USE OF THE MOON AS CALIBRATOR FOR
FIGURE-OF-MERIT MEASUREMENT OF EARTH STATIONS

Study undertaken as part of OTC R&D PROGRAM No. 4 - SATELLITE
EARTH-STATION TECHNIQUES: ASSOCIATED ANTENNA DEVELOPMENTS
AND TECHNIQUES

P.O. BOX 76,
EPPING, NSW 2121, Australia
USE OF THE MOON AS CALIBRATOR FOR FIGURE-OF-MERIT MEASUREMENT OF EARTH STATIONS

R.P. Corkish
Abstract

The Moon is gaining popularity as a source of microwave radiation for $G/T$ measurements of satellite earth stations because of its high and predictable flux density, broad-band character and visibility over a wide range of elevation angles. This report reviews existing results of measurements of the lunar flux density over the full microwave range, giving emphasis to the commercial satellite communication bands at 6/4 and 14/12 GHz, and considers the problems involved in using these results for earth-station measurements.

1 Introduction

The figure-of-merit, $G/T$, of a satellite earth station is the ratio of the antenna gain, $g$, to the system noise temperature, $t$, in kelvins. It is usually expressed in decibels per kelvin, where

$$G/T = 10 \log g - 10 \log t,$$

$$G = 10 \log(g),$$

$$T = 10 \log(t).$$

(1)

$G/T$ is a measure of the sensitivity of the earth-station.

The value of $G/T$ may be determined by separate measurements of antenna gain and system noise temperature. This method is treated comprehensively elsewhere (e.g. ref. 1) but is usually less accurate [1] than direct measurement and will not be discussed here.

Section 2 outlines the requirements for direct measurement and briefly considers the relative merits of alternative radiation sources. Variations in lunar flux density are discussed in Section 3 and published results of some experimental and theoretical determinations of brightness temperature are reviewed in Section 4. Section 5 discusses the actual and potential accuracies of brightness temperature measurements.
2 Direct Measurement of $G/T$

Direct measurement of the $G/T$ ratio involves measuring the signal-to-noise ratio resulting from the observation of a calibrated radio source. Such a source must be in the far field of the test antenna, so celestial sources, or radio “stars”, planets, the Sun, the Moon or artificial satellites are usually used.

To determine $G/T$ using a radio “star”, a measurement is made of the ratio, $y$, of the system noise plus the noise power from the source to the system noise plus the noise power from nearby cold sky. Then [2,3],

$$G/T \text{(dB K}^{-1}) = 10 \log \frac{8 \times 10^{-26} \pi k (y - 1)}{\kappa \lambda^2 S},$$

where $k$ is Boltzmann’s constant ($1.381 \times 10^{-23}$ J K$^{-1}$), $\lambda$ is the wavelength (m), $S$ is the flux density (Jy)$^1$ and $\kappa$ is a correction factor near unity that accounts for atmospheric attenuation, source shape, antenna pointing error and polarization mismatch and instrumentation effects, all of which are considered in detail for the frequency range 1 to 10 GHz in reference 4, (Section VII, A).

The use of celestial radio sources is severely limited by their flux densities and declinations [3]. Use of sources of low flux density results in a small $y$-factor, which, in turn, leads to a large relative error in the $G/T$ determination. This is shown by equation 3, taken from reference 3, which gives the maximum relative error in $g/t$ due to the relative errors in flux density and $y$-factor:

$$\frac{\Delta(g/t)}{g/t} = \frac{\Delta S(f)}{S(f)} + \frac{\Delta y}{y} + \frac{y}{y - 1}.$$  

Clearly, when the power received from the radio star is low, $y - 1 \to 0$ and $y/(y - 1) \to \infty$. Reference 3 states that “measured accuracy would be considerably reduced” for $y < 2$ dB.

Absolutely calibrated cosmic sources used for $G/T$ determinations and for the calibration of radio astronomy observations in the Northern Hemisphere are Cassiopeia A, Taurus A and Cygnus A [2,3,5]. Virgo A and Hydra A are often used for calibration of radio telescopes in the Southern Hemisphere. The flux density of each of these at 4

$^1$ Jy = $10^{-26}$ W m$^{-2}$ Hz$^{-1}$
TABLE 1. G/T minima for G/T measurement using selected cosmic sources at 4 GHz

<table>
<thead>
<tr>
<th>Cosmic source</th>
<th>Elevation from Sydney at transit (°)</th>
<th>Flux dens. (Jy)*</th>
<th>G/T minimum for 2 dB y-factor (dB K⁻¹)</th>
<th>Ref. for flux dens.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cassiopeia A</td>
<td>&lt;0</td>
<td>1067</td>
<td>35</td>
<td>3</td>
</tr>
<tr>
<td>Taurus A</td>
<td>16</td>
<td>679</td>
<td>37</td>
<td>3</td>
</tr>
<tr>
<td>Cygnus A</td>
<td>35</td>
<td>483</td>
<td>39</td>
<td>3</td>
</tr>
<tr>
<td>Virgo A</td>
<td>44</td>
<td>82</td>
<td>47</td>
<td>6</td>
</tr>
<tr>
<td>Hydra A</td>
<td>69</td>
<td>17</td>
<td>54</td>
<td>6</td>
</tr>
<tr>
<td>Moon</td>
<td>7149</td>
<td></td>
<td>27</td>
<td>12</td>
</tr>
</tbody>
</table>

*1 Jy = 10⁻²⁶ W m⁻² Hz⁻¹.

GHz is given in Table 1 and the earth-station G/T required to produce a y-factor of 2 dB at that frequency is stated for each. At 12 GHz the flux density of Virgo A is 39.56 Jy [6] so a G/T ratio of 59 dB K⁻¹ is needed to produce a 2 dB y-factor. From this it is clear that these calibrated sources are unsuitable for measurements on earth stations with low G/T values.

Table 1 also gives the elevation angle at transit (maximum elevation) from an earth-station site with longitude −33° (e.g. Sydney, Australia). The strongest calibrated star Cassiopeia A, is never visible from such a site, and radiation from Taurus A and Cygnus A would be significantly affected by the Earth's atmosphere because of their low elevations. Virgo A and Hydra A are more easily visible from the Southern Hemisphere [6], but their flux densities are relatively low. Consequentially, radio stars are unsuitable for Southern Hemisphere G/T measurements on all but very sensitive earth stations.

In order to use a satellite source [7] a saturated, unmodulated carrier signal should be obtained to ensure adequate power stability, although a method has been developed [8] to convert a frequency-modulated satellite broadcast signal to a CW signal by cancelling its modulation components. Saturated carriers are expensive and limit
the measurement to frequencies for which a transponder is available and cooperation is required with a separate well-calibrated receiving station to measure the down-link flux density. Use of geostationary satellites also severely limits the range of elevation, azimuth and polarization angles and frequencies for which $G/T$ can be measured.

The Sun is the strongest radio source in the sky but because its radiation varies greatly as a result of flare activity, its use for $G/T$ determination leads to large measurement errors [9,10].

Planets, such as Venus, may be used at frequencies greater than 10 GHz [3] but their flux densities are low, mainly because of the small angular dimensions of the planetary discs as seen from Earth. At millimetre wavelengths planets may be useful as radiation sources.

The Moon has approximately the same angular size as the Sun ($0.533^\circ$) but a much lower brightness temperature. (The brightness temperature is the temperature of a black-body radiator which would have the same monochromatic brightness at the frequency of observation [11].) Since the flux density available is proportional to the brightness temperature this means that much less power is available from the Moon. However, the lunar radiation is much more predictable than that from the Sun and is therefore more useful for earth-station measurements.

The lunar flux density has been expressed as [12]:

$$S(\phi, f) \approx 7.349 \times 10^{-18} f^2 \bar{T}(\phi, f) d^2(\phi) \text{ Jy}$$

where $\bar{T}(\phi, f)$ is the disc-average lunar brightness temperature in kelvins as a function of lunar phase ($\phi = 0^\circ$ at New Moon), $f$ is the frequency in hertz and $d(\phi)$ is the angular diameter of the Moon subtended at the antenna in degrees. The derivation of equation (4) is given in reference 12, (Appendix A). The flux density at 4 GHz is included in Table 1 for comparison with that available from radio stars. An earth station requires a minimum $G/T$ of approximately 27 dB K$^{-1}$ at 4 or 12 GHz to produce a 2 dB $\gamma$-factor when using the Moon as source.

The Moon's large angular extent leads to the need for convolution of the brightness distribution with the radiation pattern of the test antenna. The radiation intensity and its distribution vary over the lunar cycle (lunation). This will be discussed in detail.
in Section 3 below.

On the whole, the Moon has many advantages over other celestial sources for $G/T$ measurements in the Southern Hemisphere. It is a strong and predictable source of radiation over a very broad frequency range; it is easily visible from most areas and it allows measurements to be carried out over a range of elevation angles. Much interest has recently been shown in using the Moon for $G/T$ determinations [2,5,12-17], particularly for earth-stations whose $G/T$ is too low for radio stars to be used.

The main source of error involved in using lunar radiation is the uncertainty associated with the Moon's brightness temperature and its distribution over the lunar disc. A review of some past determinations of lunar brightness temperature, both experimental and theoretical, is presented below with a brief discussion of some of the discrepancies in the literature relating to them. Variations with observation frequency, with lunar phase and across the apparent disc are considered, as are the averaging effects of antennas with different beamwidths. The commercial communications satellite frequency bands at 6/4 and 14/12 GHz are of particular interest because a large number of earth stations operate in these bands. Special attention is paid to the brightness temperature at these frequencies.

3 Lunar Flux Variations

It is clear from equations (2) and (4) that uncertainties in $\bar{T}(\phi, f)$ will produce proportional errors in the final value of $G/T$. A study of systematic flux density errors in the range 1 to 10 GHz [12] concluded that the achievable error in lunar flux density is $\pm 12.8\% (\pm 0.52$ dB) linear sum error or $\pm 7.8\% (\pm 0.33$ dB) quadrature sum error. This assumes that the time-average component of the disc-average brightness temperature, $\bar{T}_0$, can be measured to an accuracy of 5% although the use of values of flux density measured in the 1950s and 1960s can be expected to result in greater error.

In order to be able to accurately predict the lunar brightness temperature, both temporal (over a 28-day lunar cycle or lunation) and spatial (over the Moon's apparent disc) variations which occur as the sub-solar point on the Moon's surface moves across the disc need to be characterised. The maximum brightness at radio frequencies
lags behind the maximum illumination by a phase shift, $\psi(f)$, which decreases with increasing frequency of observation [2,9]. The angular size of the Moon is around 0.53° and varies with the Earth-Moon distance. Allowance must be made for this source extension and its variation in all measurements using the Moon. In addition, the lunar surface is not homogeneous and variations in radio brightness occur over it [9,18], although these variations are usually insignificant at microwave and lower frequencies.

Because the lunar disc is of finite size, different receiving antennas "see" different amounts of it. A radiometer whose input comes from a very high-gain antenna measures the brightness temperature of the small section of the Moon's surface at which the main beam is pointed, while a radiometer connected to a low-gain antenna measures the average brightness temperature of the disc. In this report the brightness temperature of the centre is designated $T(\phi, f)$ and the disc-average value, $\bar{T}(\phi, f)$. A method has been derived [19] by which brightness temperature measurements using antennas of different size may be approximately compared, although the assumptions (Gaussian main lobe and no sidelobes) on which the method is based may not be valid [18]. The disc-average brightness, $\bar{T}(\phi, f)$, is the important parameter for $G/T$ measurements on earth-stations with antennas whose half-power beamwidth (HPBW) is greater than the Moon's angular size, but if both the brightness distribution across the disc and the two-dimensional radiation pattern of the antenna are known, then the two can be convolved to determine the received flux density. Sample brightness temperature contour maps of the lunar disc at 9.4 GHz [9,20] are reproduced in Figure 1.

Both $\bar{T}(\phi, f)$ and $T(\phi, f)$ vary over a 28-day lunation. For frequencies $\lesssim 20$ GHz this variation is usually assumed to be sinusoidal, although additional harmonics should be considered at higher frequencies [12]. A commonly used expression for the disc-average brightness as a function of lunar phase, $\phi$, and observation frequency, $f$, is [9,10,12]

$$T(\phi, f) = \bar{T}_0(f) \left[ 1 - \frac{T_1(f)}{\bar{T}_0(f)} \cos(\phi - \psi(f)) \right],$$

where $\bar{T}_0(f)$ is the time-average component and $T_1(f)$ is the magnitude of the first harmonic of the disc-average brightness temperature and $\psi(f)$ is the phase lag. The
Fig. 1 - Brightness temperature contour maps of the Moon at 9.4 GHz [9]. The lunar phase is $\phi$, reckoned from $\phi = 0^\circ$ at New Moon, where (a) $\phi = 168^\circ$, (b) $\phi = 194^\circ$, (c) $\phi = 230^\circ$, (d) $\phi = 262^\circ$. Temperature values are in kelvins. Error in relative values is estimated as $\pm 5\%$ and the absolute error as $\pm 15\%$. The 3 dB beamwidth of the observing antenna was $6.3^\prime \pm 0.2^\prime$. 
equation corresponding to equation (5) for disc-centre values is
\[ T(\phi, f) = T_0(f) \left[ 1 - \left( \frac{T_1(f)}{T_0(f)} \right) \cos(\phi - \psi(f)) \right], \]  
where \( T_0(f) \) is the lunation-average and \( T_1(f) \) is the magnitude of the first harmonic of the disc-centre brightness temperature. Accurate knowledge of the appropriate set of variables (either disc-average or disc-centre), and of \( T_0(f) \) or \( T_0(f) \) in particular, is necessary to quantify the lunar flux received by an antenna.

The phase lag, \( \psi(f) \), is a result of the fact that lunar radio emission is due to solar heating of dust and rocks in the Moon’s upper layers. Light is simply reflected from the surface \( (\psi = 0) \) while infrared radiation is believed to be connected with a thin surface layer and lower frequencies come from deeper rock layers [9]. Like \( \psi \), the lunation-average brightness temperature, \( T_0 \) or \( T_0 \), decreases with increasing frequency, but the phase-dependent component, \( T_1 \) or \( T_1 \), becomes much more significant at higher frequencies. Consequently the brightness temperature at low frequencies may be assumed (with an error which increases with frequency) to be constant at \( T_0 \) or \( T_0 \) over a lunation but at high frequencies the first and subsequent harmonics become a significant proportion of the average value.

4 A Review of Lunar Microwave Brightness-Temperature Observations

Historically, observations of the lunar brightness temperature have not been directed towards the use of the Moon as a radiation source for antenna measurements but towards the remote investigation of the composition of the upper layers of the Moon’s crust. The first published result for the brightness temperature at radio frequencies was by Dicke and Beringer [21] in 1946 at 24 GHz. Piddington and Minnett [22] were responsible for noting that both \( T(\phi) \) and \( \bar{T}(\phi) \) could be approximated by a sinusoidal variation (see eqns. 5 and 6).

Workers at the Research Institute of Radiophysics of the USSR made extensive measurements in the frequency range 400 MHz to 300 GHz in the late 1950s and early 1960s [23]. The results of many of these measurements have been collected and
considered in a review paper [23] in which tabulated brightness temperature values have quoted measurement errors ranging from ±2% to ±20%. The authors note that at some frequencies discrepancies between data obtained by different workers are such that the error bars for the results do not overlap. Furthermore, two results are discarded as being "patently erroneous". Many of the results which claim errors of ±3% or less were obtained by using an "artificial moon" calibration technique in which an absorbing disc was placed in the far field of the antenna [10,24]. Some doubt has been cast on this calibration method [18] because of the elaborate corrections for diffracted terrestrial radiation by the disc and because of some of the other assumptions involved.

Zheleznyakov [9] has collected and graphically presented the results of many early observations and his plots are given here as Figures 2 to 5. The results are divided into two classes: those measured with antennas whose beamwidths were less than the angular extent of the Moon (shown as open circles) and those measured with broad-beam antennas (shown as filled circles). Hence, the open circles show approximations to $T_0$ in Figures 2 and 3 and to $T_1/T_0$ in Figure 4 while the filled circles show approximations to $\bar{T}_0$ and $\bar{T}_1/\bar{T}_0$. Figure 2 shows the large discrepancies between the values of lunation-average brightness temperature measured by different researchers and clearly indicates the problem noted above in that the error bars for different measurements at the same frequency do not overlap. Measurements by Troitskii et al. [23] are considered to be more reliable and are shown separately in Figure 3. This curve indicates $\bar{T}_0$ values of 215 K at 6/4 GHz and 210 K at 14/12 GHz. These observations have recently been used as the basis for the measurement [13] of the $G/T$ ratio of a 7.3-m earth station operating at 11.7 GHz, for which a linear-sum error of ±21% (±0.8 dB) and quadrature-sum error of ±0.5 dB was claimed. For this measurement, only the lunation-average brightness temperature was considered and an error component of ±7% was included to account for phase variation.

Figure 4 records measurements of $\bar{T}_0$ and $\bar{T}_1/\bar{T}_0$, the amplitude of the first harmonic of the brightness temperature to the lunation-average and Figure 5 shows the phase, $\psi$, of the first harmonic. It is evident that the phase is not well known and this is also clear from the tabulated values in reference (23). For example, separate
Fig. 2 - Results of lunation-average lunar brightness temperature observations shown as a function of wavelength. Open circles indicate antennas with half-power beamwidths less than the Moon's angular diameter and filled circles antennas with wider beams. The wavelengths of particular interest are around 2.1/2.5 cm (14/12 GHz) and 5.0/7.5 cm (6/4 GHz). The bracketed reference numbers indicate the originators of the data as identified in reference 9 (Chap. V), from which this figure is reproduced.
Fig. 3 - Results of lunation-average lunar brightness temperature measurements made with equipment calibrated by the "artificial moon" technique [24]. All observations were made with broad-beam antennas. (Reproduced from ref. 9 as indicated in caption to Fig. 2.)
Fig. 4 - Ratio of the first harmonic of lunar brightness temperature to the lunation-average value as a function of wavelength. The open circles refer to broad-beam antennas and therefore record $T_1/T_0$ and the filled circles to narrow-beam antennas and record $T_1/T_0$. Errors in ratio measurements are usually much less than those in absolute measurements. (Reproduced from ref. 9 as indicated in caption to Fig. 2.)
Fig. 5 - Measurements of the phase lag, \( \psi \), of the first harmonic of brightness temperature behind the lunar phase, \( \phi \), as a function of wavelength [9]. (Reproduced from ref. 9 as indicated in caption to Fig. 2.)

measurements at 9.4 GHz yielded \( \psi \) values of 15°, 26°, 50° and 55°. It is, of course, pointless to know the magnitude of the first harmonic of the variation over a lunation unless a reasonable approximation can also be made to the phase. The ratios \( T_1/T_0 \) and \( \bar{T}_1/\bar{T}_0 \) obtained from Figure 4 are in the ranges 7% to 10% at 14/12 GHz and 2% to 5% at 6/4 GHz. Therefore, if the phase variation is not included or if the phase lag is not known, error components of \( \pm 10\% \) at 14/12 GHz and \( \pm 5\% \) at 6/4 GHz will be associated with brightness temperature determinations.

Kuz'min and Salomonovich [10] have collected, tabulated and graphed\(^2\) a small selection of results of lunation-average and first-harmonic measurements, but have modified the results to account for the differing beamwidths of the observing antennas and hence approximated and tabulated both the disc-average and disc-centre brightness temperatures (lunation-average and first harmonic). Reference 23 is cited but the method of obtaining the tabulated values from the data given in reference 23 is not

\(^2\)Note that their graph (Fig. 2.5) does not agree with the values in their appendix, Table 2, apparently because the scale on the ordinate is incorrect
explained, although it is reasonable to assume that a method similar to Krotikov's [19] (see Sect. 3 above) was used. Uncertainties associated with particular values are not stated for the tabulated values [10], although it is claimed that absolute values are within 5% to 10% and ratios of the first harmonic to the lunation-average are claimed to be accurate to within 2%. It is not known whether allowance was made for the additional uncertainty associated with the application of a beamwidth correction factor. A maximum error of 10° in the phase lag is claimed for the measurements below 15 GHz and 5° for those above.

Hagfors [25] is another author who has collected and tabulated the results of lunar brightness measurements made during the 1950s and 1960s. He describes the method of Krotikov [19] which may be used to compare disk-average to disc-centre brightness and uses the method to obtain curves for $T_1(f)/T_0(f)$ and $\psi(f)$.

Daywitt [12] accepts the published values [10] as the basis for a study of the errors involved in the use of the Moon for antenna measurements but states that modern measurement techniques could reduce the error in each to <$5\%$. He has fitted a curve,

$$T_0(f) \approx 207.7 + \frac{24.43}{f},$$

(7)

where $f$ is in gigahertz, to the data [10] for frequencies below 10 GHz. Using this approximation, we find have $T_0$ values of 213.8 K at 4 GHz and, extrapolating, 209.7 K at 12 GHz. The approximation

$$T_1(f)/T_0(f) \approx 0.004212f^{1.224}$$

(8)

(2% at 4 GHz and, extrapolating, 9% at 12 GHz) is given for the magnitude of the first harmonic of the disc-average brightness and at the disc-centre,

$$T_1(f)/T_0(f) \approx 0.005192f^{1.153}$$

(9)

(3% at 4 GHz and, extrapolating, 9% at 12 GHz). The phase of the first harmonic of the disc-average brightness, $\psi$ is given as

$$\psi(f) \approx \frac{43.83}{1 + 0.0109f}$$

(10)

(42° at 4 GHz and, extrapolating, 39° at 12 GHz).
Interest in the remote measurement of lunar radio brightness waned while the Apollo program was active, but it has since been revived, particularly in the USA [26]. Linsky [18, 27] used data from Apollo and elsewhere to devise a semi-empirical model for the lunar surface layers and hence obtain estimates for $T_0(f)$ over a wide range of frequencies. He compares the model against selected experimental data which were measured using antennas with angular resolution of $10'$ or less. He notes [18] that two assumptions on which the beamwidth correction factor of Krotikov [19] are based (i.e. the antenna has a main lobe of Gaussian shape and no side-lobes) may not be valid and hence restricts his comparison of theoretical and experimental values to those measurements which did not require correction for beamwidth. Reasonable agreement was achieved for the six sets of results considered (300, 91, 75, 38 and 9.4 GHz)\(^3\), but Linsky concludes "the accuracy of our method is limited by our poor knowledge of the thermal and electromagnetic properties of the mean lunar surface". His estimate for $T_0$ is graphed as a function of wavelength in Figure 6. At both 6/4 and 14/12 GHz, a value of $240 \pm 10$ K is obtained. Application of Krotikov's correction factor [19] of $0.94 \pm 0.02$ yields a $T_0$ value of $226 \pm 14$ K at both frequency bands of interest.

A group of researchers associated with the Jet Propulsion Laboratory of the California Institute of Technology have made observations at 300 GHz [29], 100 GHz [30, 31], 10.7 GHz [32], 8.15 GHz [33], 5.0 GHz [32], 2.4 GHz [29] and 2.3 GHz [32], although the results of many observations have not been published [32] and absolute calibration has not always been attempted [33]. An extensive program for high resolution mapping of the brightness temperature distribution over the lunar disc has been carried out at several frequencies. The resulting data have contributed to the formation of a detailed regolith model which is to be used for the calibration of NASA's planned Cosmic Background Orbiting Experiment (COBE) [34]. Parameters of the model were constrained by the results of measurements on lunar material brought back by Apollo missions. Figure 7(a) is the theoretical brightness temperature distribution at 8.5 GHz.

\(^3\)Both Hagfors [25, Table 3] and Linsky [18, Table II] appear to have misread the results of Low and Davidson's [28] observations at 300 GHz. The value for $T_0$ in Hagfors' table should be 213 K, which was derived from $229 \pm 10$ K, the estimated physical temperature of the lunar crust near the surface, by assuming a surface emissivity of 0.94.
Fig. 6 - An estimate [27], with estimated error region, of the lunation-average disk-centre brightness temperature, $T_0$, for a range of frequencies from 30 THz to 300 MHz. The estimate is semi-empirical in nature but is not based on any radio observations.
Fig. 7 - Theoretical (a) and experimental (b) contour maps of lunar brightness temperature at 8.5 GHz [33]. For Full Moon the experimental map was made from scans across the lunar disk with the beam of a large antenna with half-power beamwidth of 2.2° arc. The phase lag, $\psi$, of the radio "hotspot" behind the sub-solar point (disk centre at Full Moon) is evident. Relative values of contours are claimed to be within ±0.5 K but an absolute calibration was not attempted. Values are normalized to the theoretically determined disk-centre brightness temperature. (The theoretical model is also used to produce the maps in the Appendix.)
TABLE 2. Selected experimental and theoretical determinations of lunar brightness temperature.

<table>
<thead>
<tr>
<th>Reference</th>
<th>6/4 GHz</th>
<th>14/12 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\bar{T}_0$ (K)</td>
<td>$T_0$ (K)</td>
</tr>
<tr>
<td>Krotikov et al. [23]</td>
<td>215</td>
<td>210</td>
</tr>
<tr>
<td>Daywitt [12]</td>
<td>213</td>
<td>210</td>
</tr>
<tr>
<td>Linsky [27]</td>
<td></td>
<td>240</td>
</tr>
<tr>
<td>Keihm [34]</td>
<td>214</td>
<td>242</td>
</tr>
</tbody>
</table>

as produced by the model and Figure 7(b) is a map produced from observations with an antenna with 2.2' HPBW. Relative accuracy of $\pm 0.15$ K is claimed but the absolute values of Figure 7(b) have been adjusted so that the theoretical and experimental disc-centre values agree.

Stephen Keihm, of Science Applications Incorporated, has kindly provided sets of "spot" brightness temperatures across the disc as determined from the model at various phases at 4, 6 and 13 GHz. This data has been plotted as contour maps, which are given in the Appendix. These maps clearly show the shift of the radio "hotspot" along the lunar equator over a lunation. Absolute accuracy of the contours in these maps is claimed [34] to be $\pm 7$ K. Figures 8(a) and 8(b) show the variation with lunar phase of $T(\psi)$ and $\bar{T}(\psi)$ at 4 GHz, 6 GHz and 13 GHz according to the model. They show that a sinusoidal approximation is inadequate, particularly at 13 GHz. Values of $T_0$ and $\bar{T}_0$ have been calculated and are included in Table 2 for comparison with measured values from selected authors cited above.

This regolith model provides the most recent, and probably most accurate, estimation of the lunar radio brightness and hence the distributions given in the Appendix ought to be used for $G/T$ measurements, at least until independent remote measurements can be made to confirm or disprove the model.
Fig. 8 - The variation of (a) disk-centre and (b) disk-average lunar brightness temperatures at 4, 6 and 13 GHz as determined by the theoretical regolith model [32].
5 Discussion

Figures 2 to 5 clearly indicate that knowledge of the lunar brightness temperature is incomplete. Very large discrepancies exist, particularly for lunation-average temperature (Figs. 2 and 3) and for the phase of the first harmonic (Fig. 5). While measurements of the relative amplitudes of the first harmonic and of the lunation-average have achieved reasonable consistency (Fig. 4), this information is of limited use owing to the poor knowledge of the phase. Accuracy of 0.5 K (≈ 0.2%) is possible for relative measurements [33] but results with this accuracy are not generally available.

On the other hand, $T_1$ and $\overline{T_1}$ can confidently be said to vary less than ±10% over a lunation at our frequencies of interest and errors in $T_1$ and $\overline{T_1}$ are therefore less drastic than errors in $T_0$ and $\overline{T_0}$. At least one group [13] has chosen to disregard the variation of brightness with $\psi$ and simply include a ±7% term in their error analysis. An error of only ±0.18% was found [12] to be introduced into $\overline{T}$ by neglecting the second and higher harmonics at 10 GHz but observations at millimetre-wave frequencies [e.g. 10,28] indicate that a more complete model is needed at higher frequencies.

It has been claimed recently [12] that modern measurement techniques would allow $\overline{T_0}$ to be determined to an accuracy of approximately ±5% over a wide frequency range, but the body of existing results does not meet this level of precision. The measured results which do exist have relatively large uncertainties and, even then, sometimes fail to agree.

Some doubt has been cast on the hitherto accepted method [19] of translating measured values of disc brightness between disc-average and disc-centre. All generic methods of accounting for the finite angular extent of the source must make assumptions about the radiation pattern of the observing antenna and the brightness distribution across the source. In some circumstances, particular assumptions may not be accurate enough.

More accurate determinations of $G/T$ than have been obtained previously using the Moon should be possible if (1) more detailed knowledge of the brightness distribution across the lunar disc, obtainable by scanning with a narrow-beam radio telescope, and (2) the radiation pattern of the antenna-under-test are available. Convolution of
the two will allow measurements to be made on earth stations with broad beams, so long as the \( G/T \) value is high enough to enable a y-factor measurement to be made. Lunar radio brightness maps similar to those in the Appendix, but absolutely calibrated, need to be available at a wide range of frequencies and lunar phases. The regolith model [26] goes a long way towards this end but independent experimental support is required.

6 Conclusion

If the Moon is to be seriously considered as an RF source for the calibration of earth stations, a comprehensive measurement program should be undertaken to determine the values of the parameters which describe its brightness temperature at the relevant frequencies. At present situation there is an unfortunate reliance on measurements, done many years ago, whose calibration accuracy has been called into question. Modern instruments and techniques could significantly reduce the uncertainties at present associated with the use of the Moon.

In the meantime, brightness temperature distributions produced from the comprehensive regolith model should be used for measurements of \( G/T \) on earth stations with \( G/T > 27 \text{ dB K}^{-1} \) at 4 and 12 GHz. If we assume system temperatures of 100 K at 4 GHz and 250 K at 12 GHz, this \( G/T \) value corresponds to antenna gains of approximately 47 dB at 4 GHz and 51 dB at 12 GHz or antenna diameters of approximately 6.9 m at 4 GHz and 3.6 m at 12 GHz (60% aperture efficiency).

References


[6] Parkes - 85 Catalogue, CSIRO Division of Radiophysics (June 1985)


[34] Keihm, S.J., private communication, July 20, 1987
Appendix

This appendix contains 24 sets of tabulated lunar brightness temperature values as would be observed with an antenna with a very narrow beam and contour plots generated from the tables. The values were produced from an electrical/thermal model of the lunar regolith [34] and were kindly provided by Mr. Stephen Keihm of the Planetary Science Institute, Science Applications Incorporated, 283 South Lake Avenue, Pasadena, California 91101, USA. Tables and plots are provided for three frequencies, 4 GHz, 6 GHz and 13 GHz and eight lunar phases, $\phi = 0^\circ, 45^\circ, 90^\circ, 135^\circ, 180^\circ, 225^\circ, 270^\circ$ and $315^\circ$. The absolute accuracy of the brightness temperatures is claimed to be $\pm 7$ K and the relative accuracy is much better.
\[ F = 4 \text{ GHz}, \phi = 0^\circ \]
$F = 4 \text{ GHz}, \phi = 45^\circ$
<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

\( F = 4 \text{ GHz}, \phi = 90^\circ \)
F = 4 GHz, $\phi = 135^\circ$
$F = 4 \text{ GHz, } \phi = 225^\circ$
\[ F = 4 \text{ GHz}, \phi = 270^\circ \]
F = 6 GHz, ϕ = 0°
$F = 6 \text{ GHz}, \phi = 45^\circ$
$F = 6 \text{ GHz}, \phi = 135^\circ$
F = 6 GHz, $\phi = 225^\circ$
\[ F = 6 \text{ GHz}, \phi = 270^\circ \]
\[ F = 6 \text{ GHz}, \phi = 315^\circ \]
\[ F = 13 \text{ GHz}, \phi = 0^\circ \]
\( F = 13 \, \text{GHz}, \, \phi = 90^\circ \)
F = 13 GHz, \phi = 135^\circ
F = 13 GHz, $\phi = 180^\circ$
F = 13 GHz, \( \phi = 225^\circ \)
F = 13 GHz, $\phi = 270^\circ$
\[ F = 13 \text{ GHz, } \phi = 315^\circ \]