ABSTRACT: This paper describes a recent study on simulations of coupled Fracture (F) - Thermal (T) – Hydraulic (H) processes of rocks, with focus on the development a fracture mechanics code that predicts fracture initiation and propagation under thermal and hydraulic loadings. The new development is based on a numerical code FRACOD which is capable of simulating both mode I (tensile) and mode II (shear) fracture propagations that are common in rock masses. In this study, the thermal-mechanical coupling in FRACOD was developed using an indirect method based on fictitious heat sources and a time-marching scheme. The hydro-mechanical coupling in FRACOD was focused on fluid flow in explicit rock fractures using the cubic law. An explicit iteration method was used to simulate the fluid flow process in fractures and its interaction with the mechanical deformation. Two verification and applications cases have been included in the paper that demonstrate the effectiveness of the coupled functions.
gation in rock masses. The code has a unique feature and is capable of simulating both mode I (tensile) and mode II (shear) fracture propagations that are common in rock masses. The code has been applied to study borehole breakouts (Shen, 2008), the stability of large shaft and galleries (Stephansson et al, 2003), pillar spalling (Rinne et al, 2003), fundamental creep behaviour of rock samples (Rinne, 2008; Shen and Rinne, 2007), and rock indentation (Tan, 1996). This study is to advance the existing FRACOD for thermal-hydraulic-mechanical analysis of rock fractures.

FRACOD is based on the Displacement Discontinuity (DD) method, which is an indirect boundary element technique. For the benefit of coupling with FRACOD, an indirect method is also considered for simulation of the temperature distribution and thermal stresses due to internal and boundary heat sources. The indirect approaches have been found efficient in modelling poroelasticity (Ghassemi et al, 2001) and thermal-poroelasticity (Zhang, 2004) using boundary element methods.

The two-dimensional fundamental solutions for temperature and stresses induced by a continuous point heat source in thermo-elasticity are given below (Zhang, 2004; Berchenko, 1998).

\[
T = \frac{1}{4\pi k} Ei(\xi^2)
\]

\[
\sigma_{xx} = \frac{E\alpha}{24\pi k(1-\nu)} \left\{ \left[ 1 - \frac{2x^2}{r^2} \right] \frac{1-e^{-\xi^2}}{\xi^2} - Ei(\xi^2) \right\}
\]

\[
\sigma_{xy} = \frac{E\alpha}{24\pi k(1-\nu)} \left\{ \frac{2xy}{r^2} \frac{1-e^{-\xi^2}}{\xi^2} \right\}
\]

\[
\sigma_{yy} = \frac{E\alpha}{24\pi k(1-\nu)} \left\{ \left[ 1 - \frac{2y^2}{r^2} \right] \frac{1-e^{-\xi^2}}{\xi^2} - Ei(\xi^2) \right\}
\]

\[
u_x = \frac{\alpha(1+\nu)}{4\pi k(1-\nu)} r \left[ \frac{x}{2\xi^2} \frac{1-e^{-\xi^2}}{\xi^2} + \frac{1}{2} Ei(\xi^2) \right]
\]

\[
u_y = \frac{\alpha(1+\nu)}{4\pi k(1-\nu)} r \left[ \frac{y}{2\xi^2} \frac{1-e^{-\xi^2}}{\xi^2} + \frac{1}{2} Ei(\xi^2) \right]
\]

(1)

where: \( T \) – temperature (°C); \( \sigma_{xx}, \sigma_{xy}, \sigma_{yy} \) – stresses (Pa); \( u_x, u_y \) – displacements (m); \( \alpha \) - linear thermal expansion coefficient (°C); \( k \) - thermal conductivity (W/m°C); \( c \) – thermal diffusivity (m²/s); \( c = k/(\rho c_p) \); \( \rho \) - density (kg/m³); \( c_p \) – specific heat capacity (J/kg°C);

In the above equations:

\[
r = \sqrt{x^2 + y^2}
\]

\[
\xi^2 = \frac{r^2}{4ct}
\]

\[
Ei(u) = \int_{u}^{\infty} \frac{e^{-z}}{z} \, dz
\]

Equation (1) constitutes the fundamental equations to be used in all the formulations of the numerical process for F-T coupling in this paper. Because FRACOD uses 2D line elements to represent problem boundaries, we will then need to consider a line heat source solution in an infinite medium. This can be done by integrating Equation (1) over the element length. In FRACOD the integration is done numerically using ten evenly distributed points along each line element.

The basic principle of the indirect boundary element approach for thermoelastic analysis is the assumption that a fictitious line heat source exists at each element. The strengths of the line sources are unknown and should be determined based on the boundary conditions. For example, if the temperature at all boundary elements is zero, the combined effect of all the line heat sources on the boundary elements should result in a zero temperature. Once the strength of each fictitious heat source is determined, the temperature, thermal flux, and thermal-induced stresses and displacements at any given location in the rock mass can be calculated using Equation (1).

The following steps are involved in implementing the thermal function into FRACOD:

- Solve the thermal problem separately without mechanical calculations;
- Calculate the thermal stress at the centers of all boundary elements;
- Solve the same problem with mechanical load and obtain the displacement discontinuities of each element;
- Calculate the stresses and displacements at any internal point in the rock mass using the resultant displacement discontinuities. The thermal stresses and displacements need to be added to their mechanical values and they are calculated using fictitious and real heat sources.

In the coupled F-T FRACOD, the following new functions are added for thermal-mechanical analysis:

- Two types of thermal boundary conditions can be used: temperature or heat flux.
- Fractures can be treated as an internal thermal boundary. It can have constant temperature, constant heat flux or zero thermal resistance (i.e. the derivative of heat flux is constant across the fracture);
- Internal heat sources are allowed, which include both point sources and line sources in two dimensions. The internal heat sources can have variable strength.
In fractured hard rock such as granite, fluid flow occurs predominantly through explicit fractures rather than intact rock due to the low permeability of the intact rock. Fluid pressure in rock fractures may cause rock fracture movement, increase fracture aperture or even cause fracture propagation. On the other hand, fracture movement and propagation will change the fracture hydraulic conductivity and create new flow paths. The dynamic interaction between fracture mechanical response and fluid flow is critically important in studying the coupled fracture–hydraulic flow (F-H) processes.

Two fundamental approaches have been used in modelling the hydro-mechanical coupling in fractured rock medium. The first is the implicit approach, where fluid flow equations are solved together with mechanical equations for rock matrix and fractures. Most of the finite element codes designed for modeling the porous flow using Darcy’s law are based on this approach. The boundary element code used by Zhang et al. (2005) to simulate hydraulic fracturing is also based on this approach.

The second is the explicit approach, where both fluid flow and mechanical response are simulated using a time marching iteration process. The well known commercial code UDEC (Itasca, 2010) are based on this approach. Comparing with the implicit approach, the explicit approach is mathematically simple and easy to adopt complicated boundary conditions and changing model conditions. However it often requires significantly longer computational time as small time steps are required to achieve convergence for flow solution.

The explicit approach is used in this study. The mechanical calculation (including rock deformation and fracture propagation) is done using the Displacement Discontinuity (DD) method with an iteration scheme for modeling fracture propagation processes. The fracture fluid flow calculation is conducted through the time-marching iteration based on the cubic law.

3.1 Numerical considerations

The study is focused on fluid flow predominantly in rock fractures. However, leakages from fracture fluid channels to the rock matrix are also considered, see Figure 1.

During the mechanical numerical simulation using DD method, a fracture is distretised into a number of DD element. In flow calculation, each DD element is considered as a hydraulic domain and the adjacent domains are connected hydraulically (see Figure 2). Fluid may flow from one domain to another depending on the pressure difference between the two domains.

Figure 1. A flow system with dominant fracture flow and minor leakage into rock matrix.

Figure 2. Domain division for fluid flow simulation.

3.2 Iteration scheme

The solution of a coupled F-H problem can be achieved numerically using the iteration scheme shown in Figure 3, and the iteration steps are described below.

Step 1. Fluid flow occurs between fracture domains and fluid leaks into rock matrix. The fluid flow between fracture domains is calculated using the Cubic Law. The flow rate \( Q \) between two domains is calculated using Equation (2):

\[
Q = \frac{e^3 \Delta P}{12\mu l}
\]

where: \( e \) – fracture aperture; \( l \) – element length; \( \Delta P \) – fluid pressure difference; \( \mu \) = fluid viscosity.

The leakage from a fracture domain into the rock matrix is calculated using Equation 3.
Step 2. Fluid flow causes changes in domain fluid pressure. The new domain pressure due to fluid flow during a small time duration $\Delta t$ is calculated using Equation (4):

$$P(t + \Delta t) = P_0 + K_w Q \frac{\Delta t}{V} - K_w Q_{\text{leak}} \frac{\Delta t}{V}$$

where: $K_w$ – fluid bulk modulus; $V$ – domain volume; $\Delta t$ – time step.

Step 3. Change in fluid pressure causes fracture deformation. The fracture deformation is calculated using the DD method where the new fluid pressures in fracture domains are the input boundary stresses.

Step 4. Fracture deformation changes the domain volume, and hence change the fluid pressure in domains. The new domain pressure is calculated using Equation (5)

$$P'(t + \Delta t) = P(t + \Delta t) - K_w \frac{\Delta e}{V} \cdot l$$

The new domain fluid pressures are then used to calculate the flow rate between domains in Step 1. Steps 1 to 4 are iterated until the desired fluid time is reached and a stable solution is achieved.

4 VERIFICATION AND APPLICATION EXAMPLES

In order to demonstrate the capability of the new coupled F-T and F-H functions in FRACOD, two examples concerning thermo-elasticity and hydraulic fracturing have been considered and discussed below.

4.1 Cooling fractures in borehole wall

Let’s consider a borehole with radius $r = 0.1$ m in a geothermal reservoir with an in situ rock temperature $T_0 = 200^\circ$C. The borehole wall is cooled by drilling fluid and maintained at temperature $T_w = 80^\circ$C. The mechanical and thermal properties used in this simulation are listed below.

- Thermal conductivity: $k = 10.07$ W/m$^\circ$C
- Specific heat: $C_p = 790.0$ J/(kg$^\circ$C)
- Linear thermal expansion coeff.: $\alpha = 2.4 \times 10^{-5}$ /$^\circ$C
- Young’s modulus: $E = 37.5$ GPa
- Poisson’s ratio: $\nu = 0.25$
- Tensile strength: $\sigma_t = 12.5$ MPa
- Cohesion: $c = 33$ MPa
- Internal friction angle: $\phi = 33^\circ$
- Fracture Mode I toughness: $K_{IC} = 1.5$ MPa m$^{0.5}$
- Fracture Mode II toughness: $K_{IIc} = 3.0$ MPa m$^{0.5}$
- In situ stresses: $\sigma_{xx} = \sigma_{yy} = 10$ MPa

The numerical model for this problem has 60 constant elements on the borehole boundary. Six different cooling times were considered: $10^2$, $10^3$, $10^4$, $10^5$ and $10^6$ seconds. The modelled temperature distribution is shown in Figure 4 and the thermal stresses are shown in Figure 5.

![Figure 4. Cooling of a borehole. Rock temperature variation from hole boundary into the region at different times.](image)

![Figure 5. Cooling of a borehole. Thermal-induced stresses at different times.](image)
4.2 Hydraulic fracturing in rock mass with preexisting fractures

A typical case of hydraulic fracturing is simulated to demonstrate the effectiveness of the coupled F-H function in FRACOD. A borehole is drilled in a rock mass with several isolated pre-existing fractures (Figure 7a). A high fluid pressure is then applied in the borehole to propagate the existing fractures.

The key mechanical and fluid properties used in this model is listed below.

Fracture Mode I toughness: $K_{IC} = 1.5 \text{ MPa m}^{0.5}$

Young’s modulus: $E = 37.5 \text{ GPa}$

Poisson’s ratio: $\nu = 0.25 \text{ GPa}$

Fracture residual aperture: $c_r = 10 \mu\text{m}$

Fracture initial aperture: $c_0 = 10 \mu\text{m}$

Bulk modulus of fluid: $K_w = 2 \text{ GPa}$

Intact rock hydraulic conductivity: $1.0 \times 10^{19} \text{ m/s}$

Borehole fluid pressure: $P = 5 \text{ MPa}$

Initial pore pressure: $P_0 = 0 \text{ MPa}$

In situ stresses: $\sigma_{xx} = \sigma_{yy} = 1.0 \text{ MPa}$.

The simulated fracture propagation pattern after 0.15 seconds is shown in Figure 7. The fluid pressure distribution in the fractures is shown in Figure 8.

In this model, the fluid pressure drives the short fracture at the borehole wall to propagate toward the preexisting fractures and eventually coalesce with these fractures. The pre-existing fractures then propagate under the high fluid pressure. It is noticed that the tips of the propagating fractures appear to be well ahead of flow fronts in the fracture because the stress wave is much faster than the speed of fluid flow.
5 CONCLUSIONS

A fracture – thermal coupling (F-T) function and a fracture - fluid flow coupling (F-H) function have been recently developed using FRACOD as the basic code. The F-T coupling was achieved using the indirect method, namely the fictitious heat source method. The coupled FRACOD can now simulate thermal induced stress, displacement and fracture initiation and propagation in intact or pre-fractured rock masses.

The F-H coupling was achieved using an explicit approach and it employs a time-marching iteration scheme for both fluid flow and fracture propagation. The new function is capable of simulating fluid flow in complex fracture networks and fracture movement and propagation driven by fluid pressure. A simple mechanism of fluid leakage into intact rocks has also been implemented.

Two application cases using the coupled FRACOD have been described. One is borehole cooling in a geothermal reservoir. The other is a simulation of hydraulic fracturing processes in a rock mass with pre-existing fractures. The results demonstrate the effectiveness of the new functions in predicting rock fracturing processes due to thermal and hydraulic forces.

REFERENCES


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