THREE DIMENSIONAL MODELLING OF LAVA FLOW USING SMOOTHED PARTICLE HYDRODYNAMICS

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Abstract

The paper describes the use of the grid-free Smoothed Particle Hydrodynamics (SPH) method to investigate lava flow from volcanic eruptions using real three dimensional topography in the form of Digital Terrain Models (DTM). Heat transfer resulting from conduction and radiation and solidification of the lava modelled via a variable viscosity are coupled to the fluid flow solution. Simulations show that the run-out distance and the nature of the lava flow are affected significantly by the lava viscosity and that this is dependent on the scale of the volcanic eruption, with solidification effects strongest on the smallest scale. SPH appears to be a highly effective technique for predicting lava flow with very good representations of the fluid free surface, close interaction with the complex topography, easy inclusion of the thermal and solidification effects leading to very plausible flow predictions. The pile-up of the lava at the front as it solidifies and the subsequent deceleration of the flow are easily modelled by SPH.

Keywords

Smoothed Particle Hydrodynamics, Three-dimensional modelling, Lava flow modelling, Digital Terrain Model.
**Introduction**

Although most volcanic lava flows do not result in loss of human life, they can potentially cause enormous damage to property. Lava flows can bury homes and agricultural land under tens of meters of hardened black rock. People are rarely able to use land buried by lava flows or sell it for more than a small fraction of its previous worth. Typical examples of lava flows are from the Kilauea and Mauna Loa Volcanoes in Hawaii (USGS, 2005).

The speed at which lava moves across the ground depends on several factors, including (a) type of lava erupted and its viscosity; (b) steepness of the ground over which it travels; (c) whether the lava flows as a broad sheet, through a confined channel, or down a lava tube; and (d) rate of lava production at the vent.

The average speed of some lava flows can be as high as 3 m/s, an example being the Hawaiian lava flow at Mauna Loa (USGS, 2001). The maximum flow speed can however be quite a bit higher close to the source of eruption and depends on the slope of the terrain and type of lava. Speeds can be of the order of 15 to 20 m/s as reported by Komorowski et al. (2003) for the Mt. Nyiragongo volcano in the Democratic Republic of Congo.

Solidification of the lava is very important to the overall flow prediction since the viscosity of most lava increases exponentially with decreasing temperature as it solidifies (Klingelhöfer et al., 1999). The increase in viscosity in turn reduces the speed of the lava flow dramatically with speeds declining to as low as 0.1 to 0.2 m/s and eventually stopping.

In order to estimate the amount of damage that can be caused by a lava flow, it is useful to be able to predict the size and extent of such flows. Numerical simulation is a good tool to examine such issues. With such simulations, one can explore various eruption scenarios.
and these can specifically be used to estimate the extent of inundation area, the time required for the flow to reach a particular point (run-out distance) and resulting morphological changes. Klingelhöfer et al. (1999) used a 3D finite element model to simulate a simple three dimensional flow in a model lava tube. Itoh et al. (2000) used a finite difference method and a two-dimensional depth integrated shallow water equation to simulate pyroclastic flow in a real topography. Due to the two dimensional nature of the simulation, effects such as the pile-up of material and the resultant reduction in speed of the flow front cannot be accurately predicted by such models. Thermal effects were not included in the model by Itoh et al. (2000) although such effects can be extremely important in such flow scenarios. The work by Miyamoto and Sasaki (1997), Crisci et al. (2003) and Crisci et al. (2004) are examples of lava flow simulations using a cellular automata method. This method does not solve the full Navier-Stokes equations for fluid flow but is based on the assumption that local interactions of constituent parts finally evolve into a valid macroscopic solution. The rule base used for determining these local interactions can have a significant influence on the macroscopic solution. As an alternative, we propose the use of a method called Smoothed Particle Hydrodynamics (SPH), which allows fully three dimensional free surface simulations of lava flows to be performed on realistic topography, including heat transfer and solidification effects.

SPH was originally developed in the 1970’s to solve compressible astrophysical problems, (Gingold and Monaghan, 1977). Many applications involving free surface flows have been solved using the SPH method since then. Some examples include bursting of a dam and generation of a wave in two dimensions (Monaghan, 1994), high pressure die-casting and ingot casting of aluminium (Cleary, 2004, 2006), simulation of stirred systems (Prakash et al, 2007) and simulation of liquid sloshing (Guzel et al., 2005). Three dimensional
simulations of geophysical flows such as dam-breaks (Prakash et al., 2001 and Cleary et al. 2010) and tsunamis (Debroux et al., 2001) have also been carried out successfully with this method. These are summarised in Cleary and Prakash (2004). The natural advantages that SPH has for modelling complex free surface flow mean it is also an ideal method for modelling volcanic lava flows. The prediction of these flows is more complicated because of the need to include additional physics for processes such as:

- Ejection of liquid magma from below the ground surface level through a lava conduit in the base of the crater;
- Conductive heat transfer from the magma to the ground;
- Cooling of the magma through radiative heat losses to the air; and
- Solidification of the magma as it cools.

This study demonstrates the use of SPH as an effective simulation tool for volcanic lava flow and examines the effect of radiative and conductive cooling with a change in the scale of the volcano. The differences in the flow pattern and run out distance are illustrated for cases with and without solidification. The presence of heat transfer and solidification coupled with fluid flow on real topography in three dimensions in the model is expected to allow the behaviour of lava flows to be better predicted, due to the inclusion of more of the critical physics. This in turn will enhance the prediction of the impact of volcanic flows on surrounding life and property.

Qualitative information about run out distances and the nature of lava flow is available from sources such as Komorowski et al. (2003). This is however not sufficient for the
purposes of validation. Detailed validation will be carried out when good quantitative data becomes available for such flows.

The SPH Methodology

The SPH method used in the present simulations follows the same principles as outlined in Monaghan (1992) and modified in Cleary et al. (1998). The interpolated value of a function $A$ at any position $\mathbf{r}$ can be expressed using SPH smoothing as:

$$A(\mathbf{r}) = \sum_b m_b \frac{A}{\rho_b} W(\mathbf{r} - \mathbf{r}_b, h)$$

(1)

where $m_b$ and $\mathbf{r}_b$ are the mass and position of particle $b$ and the sum is over all particles $b$ within a radius $2h$ of $\mathbf{r}$. Here $W(\mathbf{r}, h)$ is a $C^2$ spline based interpolation or smoothing kernel with radius $2h$, that approximates the shape of a Gaussian function but has compact support. The gradient of the function $A$ is given by differentiating the interpolation equation (1) to give:

$$\nabla A(\mathbf{r}) = \sum_b m_b \frac{A}{\rho_b} \nabla W(\mathbf{r} - \mathbf{r}_b, h)$$

(2)

Using these interpolation formulae and suitable finite difference approximations for second order derivatives, one is able to convert parabolic partial differential equations into ordinary differential equations for the motion of the particles and the rates of change of their properties.

**Continuity equation:**

From Monaghan (1992), we represent the SPH continuity equation as:

$$\frac{d\rho_a}{dt} = \sum_b m_b (\mathbf{v}_a - \mathbf{v}_b) \nabla W_{ab}$$

(3)
where $\rho_a$ is the density of particle $a$ with velocity $v_a$ and $m_b$ is the mass of particle $b$. We denote the position vector from particle $b$ to particle $a$ by $r_{ab} = r_a - r_b$. $W_{ab} = W(r_{ab}, h)$ is the interpolation kernel with smoothing length $h$ evaluated for the distance $|r_{ab}|$. This form of the continuity equation is Galilean invariant (since the positions and velocities appear only as differences), has good numerical conservation properties and is not affected by free surfaces or density discontinuities. The use of this form of the continuity equation is very important for predicting free surface flows of the type found in volcanic lava flows.

**Momentum equation:**

The SPH momentum equation used here is:

$$
\frac{d\mathbf{v}_a}{dt} = \mathbf{g} - \sum_b m_b \left[ \left( \frac{P_b}{\rho_b^2} + \frac{P_a}{\rho_a^2} \right) - \frac{\xi}{\rho_a \rho_b (\mu_a + \mu_b)} \frac{\mathbf{v}_{ab} \cdot \mathbf{r}_{ab}}{r_{ab}^2 + \eta^2} \right] \nabla W_{ab}
$$

(4)

where $P_a$ and $\mu_a$ are pressure and viscosity of particle $a$ and $\mathbf{v}_{ab} = \mathbf{v}_a - \mathbf{v}_b$. Here $\xi$ is a factor associated with the viscous term as described in Cleary (1996), $\eta$ is a small parameter used to smooth out the singularity at $r_{ab} = 0$ and $\mathbf{g}$ is the gravity vector.

The first two terms involving the pressure correspond to the pressure gradient term of the Navier-Stokes equation. The next term involving viscosities is the Newtonian viscous stress term. This form ensures that stress is automatically continuous across material interfaces and allows the viscosity to be variable or discontinuous. This is important to give accurate velocity predictions for solidifying materials where the viscosity gradient can be very sharp.

**Energy equation:**

A form of the SPH heat equation based on internal energy was developed in Cleary and Monaghan, (1999). This was converted to an enthalpy formulation for use with solidifying
materials (Cleary et al., 1998). The following form of the energy equation was used for the present simulations:

\[
\frac{dH_a}{dt} = \sum_b 4m_b \rho_a \rho_b \frac{k_a k_b}{k_a + k_b} r_{ab} \nabla_a W_{ab} r_{ab}^2 + \eta^2 + \Phi_a \tag{5}
\]

where the summation corresponds to heat conduction and \(\Phi_a\) is the rate of cooling due to radiative heat losses if the particle \(a\) is part of a free surface (solid or liquid) or is conductive if adjacent to the cold basal rock. The enthalpy per unit mass is:

\[
H = \int_0^T c_p(\theta) d\theta + L [1 - f_s(T)] \tag{6}
\]

c\(_p\) is the temperature dependent specific heat at constant pressure, \(L\) is the latent heat and \(f_s(T)\) is the volume fraction of material that is solid at temperature \(T\). \(k_b\) is the conductivity, \(\rho_b\) is the density and \(T_{ab} = T_a - T_b\).

The radiative heat loss per unit mass and per unit area is given as:

\[
\Phi_a = -\varepsilon \sigma T_a^4 \tag{7}
\]

where \(\varepsilon\) is the emissivity of the surface and \(\sigma = 5.67 \times 10^{-8}\) is the Stefan-Boltzmann constant. The basal rock is kept at constant temperature so forms an isothermal boundary condition on the bottom of the flowing lava. The same conductivity is used for basal rock as is used for the lava.

**Equation of state:**

Since the SPH method used here is quasi-compressible one needs to use an equation of state, giving the relationship between particle density and fluid pressure. This relationship is given by the expression:
\[ P = P_0 \left[ \left( \frac{\rho}{\rho_0} \right)^\gamma - 1 \right] \]  \hspace{1cm} (8)

where \( P_0 \) is the magnitude of the pressure and \( \rho_0 \) is the reference density. For lava (which is a very stiff fluid) we use \( \gamma = 7 \). The pressure calculated from equation (8) is then used in the SPH momentum equation (4) to predict the particle motion. The pressure scale factor \( P_0 \) is given by:

\[ \frac{\gamma P_0}{\rho_0} = 100V^2 = c_s^2 \]  \hspace{1cm} (9)

where \( V \) is the characteristic or maximum fluid velocity. This ensures that the density variation is less than 1% and the flow can be regarded as incompressible.

The time stepping in this code is explicit and is limited by the Courant condition modified for the presence of viscosity,

\[ \Delta t = \min_a \left\{ 0.5h \left/ \left( c_s + \frac{2\xi\mu_a}{h\rho_a} \right) \right. \right\}, \]  \hspace{1cm} (10)

where \( c_s \) is the local speed of sound.

**Simulation Setup**

Simulations of the volcanic lava flow were carried out using realistic three dimensional topography, obtained from the US Geological Survey (USGS, 1998), and shown in Figure 1. The sample topography used was that of the Triunfo Pass region located in the north of the state of California in the USA. The topographic data was in the form of a Digital Terrain Model (DTM) with a resolution of 10 m. A DTM is a digital file consisting of
terrain elevations for ground positions at regularly spaced horizontal intervals. The DTM was scaled down to three different ratios namely 1:10, 1:33 and 1:100 to allow us to examine the effect of the scale of the volcanic eruption on the nature of the resulting lava flow. Lava conduit diameters of 29.0 m (large scale eruption), 8.7 m (medium scale) and 2.9 m (small scale) respectively were chosen for the three topography scales.

The DTM data was used to generate SPH boundary particles, with a spacing of 1.0 m for the large scale, 0.33 m for the medium scale and 0.1 m for the small scale, which represent the topography in the SPH simulations. The fluid particle spacing in each case was chosen to be 1.3 times the boundary particle spacing. The SPH interpolation length is then set as 1.2 times the fluid particle spacing. This resulted in approximately 800,000 boundary particles and between 400,000 and 500,000 fluid particles for each simulation.

Figure 2 shows a schematic of the simulation setup with the volcanic crater generated close to one of the peaks of the mountainous terrain. The lava source is a circular region which was placed between 0.3 m (for the small scale) to 3.0 m (for the large scale) below the bottom of the crater and which generates a continuous upward stream of lava.

Solidification was modelled using a variable viscosity model from Klingelhöfer et al., (1999) with the viscosity varying as:

$$\mu = A_\mu \exp \left( \frac{E_\mu}{R_G T} \right)$$

(11)

where $A_\mu$ is an empirical constant for lava viscosity, $E_\mu$ the activation energy of the viscous flow and $R_G$ is the gas constant (8.31 Jmol$^{-1}$K$^{-1}$).
Material properties of Hawaiian basaltic lava were chosen (Klingelhöfer et al., 1999) with $A_{\mu}=5.58 \times 10^{-6}$ and $E_{\mu}=1.88 \times 10^{5}$ J since they are typical of many basaltic rocks occurring in volcanic regions. The average density and specific heat of the lava are 2700 kg/m$^3$ and 730 J kg$^{-1}$ K$^{-1}$ respectively. The eruption temperature of the lava at the point of discharge from the crater was chosen to be 1500 K (Fagents et al., 1999) with the ground temperature being 300 K. At 1500 K, the viscosity of the lava (as given in equation (11)) is about 20 Pa s (which is very free flowing). Based on equation (11), a decrease in temperature to about 1200 K gives a viscosity of about 860 Pa s (an increase by a factor 43) demonstrating that the lava becomes significantly viscous with even moderate decreases in temperature.

Figure 3 shows the steps involved in simulating the lava flow with SPH from problem setup to visualisation consisting of:

- Creation of a volcanic crater in the DTM and conversion of the topography into an SPH boundary representation.
- Creation of the lava source in the SPH simulation at the bottom of the crater.
- Assigning of material properties to the ground and lava.
- Running of the simulations.
- Extraction of relevant quantitative information such as run-out distance of the lava and position of the lava front, and
- Visualisation of lava flow by creating surface meshes from the SPH particle data in order to provide more realistic representation of the fluid flow.

The simulations took approximately 2 weeks to run on a single processor DEC Alpha XP1000, 500 MHz machine. On modern day machines with a standard 3.6 GHz Intel
Xeon processor these simulations will take only 2 days to run on a single processor. Using our current parallel SPH solver the compute time for these simulations would be around 8 hours on an 8 core machine using 3 GHz Intel Xeon processors.

**Scaling Rules for Comparison**

In order to isolate the scale dependent effects of the heat transfer and solidification we wish to compare lava flows at the three different sizes. To do this we need to consistently scale the lava ejection speed or exit velocity, the run-out distance and run-out time. This ensures the following:

- The height reached by the exiting lava for all three cases is essentially the same.
- The run-out distances and run-out times are consistently normalised.

The velocity scaling was done by equating the ratio of the kinetic energies at the point of ejection to the ratio of their diameters as follows:

\[
\frac{v_{l1}^2}{v_{l2}^2} = \frac{d_{c1}}{d_{c2}}
\]  

(12)

where \(v_{l1}\) and \(v_{l2}\) are the speed of lava for different conduit diameters \(d_{c1}\) and \(d_{c2}\) respectively. This ensures that the maximum ejection height scales with the spatial dimension of the lava jet and will allow comparison of the normalised run-out distances enabling identification of the scale dependent effects of solidification.

Using equation (12) and assuming typical lava exit speed of 10 m/s for a conduit diameter of 2.9 m (Bluth and Rose, 2004) exit velocities, given in Table 1, were used for the three different volcano sizes simulated.

The scaling of distances between cases is done normalising by the ratio of spatial scales:
\[ d_{\text{runout}2} = d_{\text{runout1}} \times \frac{d_{e2}}{d_{e1}} \]  

(13)

where \( d_{\text{runout}} \) represents the run-out distance of each case, which is defined as the maximum distance that lava has travelled from the lava conduit in any direction.

The scaling for time is chosen to be consistent with the velocity scaling in equation (12).

Defining the average run-out speed as:

\[ u_{\text{runout}} = \frac{d_{\text{runout}}}{t_{\text{runout}}} \]

where \( t_{\text{runout}} \) represents the run-out time and using these velocities in equation (12) gives:

\[ \frac{u_{11}^2}{u_{12}^2} = \frac{d_{\text{runout1}}^2}{d_{\text{runout2}}^2} \frac{l_{\text{runout1}}^2}{l_{\text{runout2}}^2} = \frac{d_{e1}}{d_{e2}} \]

(14)

Using the distance scaling from equation (13) and re-arranging we obtain the appropriate scaling for the time to enable comparison of the different cases. This is:

\[ t_{\text{runout2}} = t_{\text{runout1}} \sqrt[3]{\frac{d_{e2}}{d_{e1}}} \]

(15)

Three dimensional lava flow on real topography

Figure 4 and 5 show aerial and oblique front views of the flow of lava for a small scale eruption from a 2.9 m diameter conduit using the coupled thermal/solidification/fluid flow SPH model described earlier. The colour of the lava in the figures indicates the temperature and viscosity of the lava. Red shows high temperature (1500 K) or low viscosity (20 Pa s) and black shows low temperature (1000 K) and high viscosity (37 kPa s). The ground, which is maintained at 300 K, is coloured from black (for high altitudes) through to grey (for low altitudes) in order to show the topography. The surrounding sea is flat and coloured blue. Note that for these simulations the sea is assumed to be a solid at
the same temperature as the ground and no attempt has been made to model the complex interactions between the lava and the sea water. In future work, these will be considered.

Early on (frames a and b of Figures 4 and 5) the lava is seen to be erupting out of the crater at a high velocity (the exit speed of the lava is 10 m/s for this case) and forms a relatively circular dome that rises up and begins to fall back to the ground due to gravity. At this stage there is very little cooling and almost the entire mass is still at around 1500 K. In frames c and d the lava has reached the ground and can be seen to be spreading across the topography, inundating the nearby valleys and low lying regions of this mountainous terrain. The lava remains very hot and has lost very little of its momentum in this period.

In frames e and f the lava has almost fully inundated the nearby valleys and is approaching the sea. The speed of the lava front has started to decline due to the increasing drag from the ground. The extent of the decline is heavily influenced by the increase in the viscosity of the lava which is controlled by the rate of cooling. The rate of cooling is strongly affected by the surface area of the lava. The area of free surface determines the radiative heat losses and the basal area determines the rate of conductive heat loss to the rock below. As the lava runs-out its exposed area increases rapidly leading to progressively faster heat loss. Rapid flow also enhances heat loss since new hot lava is more frequently exposed leading to higher heat fluxes from these surfaces. This effect can be seen from frame f where the lava front, which has just started moving onto the sea, has become black in colour (representing material that is between 1300 and 1000 K and is extremely viscous with a viscosity range of 200 to 37,300 Pa s).
Frames g and h show the further progression of the lava across the sea albeit at a much lower velocity. From these later frames it can also be seen that the lava front has started folding onto itself forming a thick mass of semi-solid material. The movement of the lava is now restricted by this more viscous front. From the last frame (frame h) of Figure 4, the lava on the valley at the top right corner is now substantially solidified close to the edges where it has been stagnant for some time.

Figure 6 shows a close-up of the lava flowing onto the cold plane representing the sea surface and its pile-up as it solidifies due to cooling. In the first frame the lava has started spreading on the surface and creates a single dark, relatively cool and viscous lava layer. In the second frame (14 s) substantial additional lava flows into this region but there is only modest sideways spread of the dark mass. The majority of the new lava arriving in this region builds up on top of the previously deposited dark almost solid base.

The pile-up of the relatively cold lava is seen more prominently in frames 3 and 4 of Figure 6. There is again only small horizontal movement of the extremes of the cold lava mass and all the material that has flowed into the sea accumulates in multiple layers of solidified rock on top of each other.

The overall flow pattern, the reduction in lava speed due to solidification enhanced drag from the ground and the pile-up of the lava due to solidification are all reasonable for these types of lava flows.

**Effect of Scale of Eruption on Lava Run-Out**
In order to assess the effect of the scale of the volcanic eruption on run-out distance, three different conduit diameters of 2.9, 8.7 and 29.0 m were used with exit velocities as given in Table 1. For comparison, the run-out distance and run-out time were normalised according to equations (13) and (15).

Figure 7 shows the comparison of the lava flow for the three different eruption sizes with frames in the first column for the large eruption, the second column for the medium one and the last column for the small volcano. In the first frame all three flows look similar with the lava flowing out of the conduits in a dome like pattern and just beginning to flow onto the valleys in the 3D terrain. In the second frame the lava has flowed through the ridges in the valleys of the mountains and has begun flowing onto the flat sea and in all three cases is still quite similar. There are however, subtle differences with slightly shorter and thinner run-out along each branch of the lava flow in the medium scale case and slightly shorter run-out again for the small case.

The effect of rapid cooling of lava for the smaller volcano is seen much more clearly in frames 3 and 4, where the lava in the right column has started turning black and has become more viscous as it reaches the sea surface. This in turn leads to a rapid deceleration of the flow and pile-up of the lava as described in the previous section. For the medium size volcano (middle column), the lava is still substantially free flowing (red) but there is clear retardation in the progress of the flow along each branch of the flow compared to the large scale volcano.

The shape of the lava front is markedly different for the smallest volcano compared to the two larger sizes; the smaller one develops semi-circular patterns as the viscous lava slows
down (as seen in frames 3 and 4). The two larger volcanoes do not show any such structural change in the flow pattern with the lava still fanning out in triangular shapes as it leaves the valleys in the mountains and reaches the sea surface. The heat transfer and solidification has only slowly and mildly thickened the flow for the medium scale case without changing the basic nature of the flow.

The reason for the much faster overall cooling and solidification for the smaller volcanic lava flows is that the rate of cooling is broadly proportional to the surface area to volume ratio of the lava, which increases linearly with decreasing volcano size. This results from the heat flux lost from the lava being dependent on the surface area (by radiation from the free surface and by conduction to the basal rock), while the amount of heat available to be lost depends on the volume of lava. The average rate of heat loss for each part of the lava therefore depends on the surface to volume ratio. For a smaller volcanic flow, this means that a larger fraction of the lava is exposed to cooling and the distances that the heat needs to conduct within the lava flow are shorter leading to faster effective internal heat transfer and therefore more efficient cooling.

Figure 8 shows the (normalised) run-out distance against (normalised) time for the three cases. This is calculated as the furthest radial distance of the lava flow from the originating conduit. In all cases, the early behaviour is parabolic with increasing rate of run-out. This reflects the collapse of the initial cone of lava erupting from the conduits and its acceleration down the steep sides of the mountain. At this early stage there are no differences because the thermal effects have had negligible impact at this time.
For the large case, the run-out continues essentially linearly for several seconds. The drag on the lava is being balanced by the conversion of gravitational potential energy to kinetic energy as the lava flows down the mountain slopes. The subtle differences in the slope represent the effect of changes in terrain slope. At around 8 time units, there is a small but nonetheless noticeable decrease in the slope of the run-out as the lava flows onto the plane representing the sea. Since this is horizontal, there is no further supply of kinetic energy to the lava. This continues linearly till the edge of the computational domain (at distance 70 in the scaled units used for comparison).

For the medium scale volcano, the run-out tracks that of the large volcano closely until around a time of 8, with only a slight retardation from the cooling. The later stages of the lava flow show reasonable divergence of the run-out curves with a much shallower slope between times 8 and 16. This reflects the increasing effect of the higher viscosity produced by the higher cooling of this size of volcanic flow. The time required to reach the distance of 70 m has increased by around 40%.

The effect of the cooling induced viscosity increase is much more marked for the smallest scale of volcano considered. The run-out curves begin to clearly diverge after only 3 time units. The retardation over the first 8 time units (when the lava reaches the sea) is around 15%. After this, the divergence is substantial with the run-out curve bending over and flattening out to a limit value of 53. This reflects the full solidification of the leading material and the pile up of following lava on top of this earlier material so that there is no further extension of the run-out distance. This leads to significant divergence of the small scale flow from even the medium scale one. It is clearly seen from this figure that the
Effect of the cooling is strongest after a reasonable amount of run-out when the area of the lava (and therefore the rate of cooling) have increased significantly.

**Effect of Variable Viscosity on Lava Run Out**

In order to further explore the effect of solidification on run-out distance, a simulation on the smallest volcano with a conduit diameter of 2.9 m was performed with no heat transfer so that the viscosity was constant at 20 Pa s, which is the free flow viscosity of the Hawaiian basaltic lava at a temperature of 1500 K. The exit velocity for both simulations was again 10 m/s.

Figure 9 shows a comparison of the lava flow for the constant and variable viscosity cases. Early in the eruption (at 5 s) the two simulations show very similar behaviour as the lava leaves the crater. At this early stage the lava has not yet reached the ground and thus has insignificant means of losing thermal energy to the surroundings.

By 10 s, lava from the eruption crater has spread out into the valleys and in both simulations has started moving onto the surrounding sea. For the constant viscosity case the lava front is more fragmented and has run-out moderately further. In the variable viscosity case, the fluid close to the sea has started cooling down and has therefore become more viscous, is thicker and moving more slowly and remains contiguous.

Differences in the flow patterns become significant at 15 s. The constant viscosity case has lava moving more rapidly and spreading out onto the sea in the form of a relatively broad but thin fan pattern from each of the inundated seaward valleys. The variable viscosity
case on the other hand has fluid fronts on the sea that are semi-circular in shape. The lava flowing from the valleys at later times has been significantly cooled and just piles up on top of the earlier solidified layers of lava.

By 20 s the differences have become very pronounced with the constant viscosity lava now spreading across a broad region of sea with additional lava from the volcano still feeding down into the sea and up along the landward facing valleys. The lava deposits in the valleys are much wider in this case as the low viscosity lava is free to flow sideways (essentially like water and fill the valleys with relatively flat top surfaces). The variable viscosity lava front on the other hand has virtually stopped extending into the sea as additional lava flows down onto the previously solidified lava layers. In the elevated valleys the lava deposits broaden slightly but by far less than for the constant viscosity case. This indicates that the later arriving lava in the valleys is also flowing centrally in the valleys, filling them with thicker and more rounded deposits concentrated more in the middle of the valleys. Note that the three dimensional nature of the simulation provides an ability to determine the fraction of lava flowing into different sections of the terrain.

Figure 10 shows a quantitative comparison of the run-out distance versus time between the two cases. With a variable viscosity, the run-out increases almost linearly with time until about 8 s. After this the flow decelerates sharply due to the rapidly increasing viscosity of the lava as a consequence of its cooling. By about 12 s the lava has essentially stopped its outward progression at around 53 m from the volcano conduit. Conversely, for the constant viscosity case the lava front continues to move almost linearly until the lower boundary of the computation volume is reached at about 12 s. The difference in the flow pattern and run-out distance clearly demonstrates that the thermal cooling and
solidification has a significant impact on small scale lava flows. This strong sensitivity of the flow behaviour means that it is very important to include these effects and to use suitable viscosity versus temperature relationships for the solidification effects in order to predict reasonable estimates of lava run-out distances. It also demonstrates the ability of the SPH method to include such variable viscosity models with relative ease.

**Conclusions**

The three dimensional flow of lava as it erupts from a crater at high speed, spreads out across the surrounding mountain valleys and eventually onto a cold sea surface was simulated for a range of volcano sizes. This was made feasible by using a grid-free fluid simulation technique called Smoothed Particle Hydrodynamics (SPH). The method involves solving the full Navier-Stokes equations coupled to heat transfer due to surface conduction and radiative losses and solidification effects modelled via a temperature dependent viscosity.

The use of realistic topography in three dimensions allows important features of volcanic lava flow to be captured in ways that are not possible using either more traditional two dimensional flow simulations or using simplified topographies. These include:

- Lava inundation of the valleys and low lying regions including branching of the flow on the mountainous terrain.
- Lava flow up-hill into the landward facing valleys and down-hill into the sea and the ability to determine the fraction of lava flow into these different sections of the terrain.
- Fanning out of the lava into almost triangular or semi-circular patterns depending on the volcano size as it spreads out into the sea.
• Changes to the slope of the run-out distance curve due to variations in potential energy supplied to the lava by the changing elevation produced by the strongly varying terrain.

• Reduction in lava speed due to solidification enhanced drag from the ground, and

• Pile-up of the lava into multiple layers due to solidification at lava fronts.

The scale of the volcanic lava flow strongly affects the importance of heat transfer from the lava to the environment. This in turn strongly affects the extent and nature of lava run-out. The key factors governing these effects are:

• The surface area increases rapidly as the lava spreads out thereby increasing the rate of cooling and solidification.

• The surface to volume ratio of the lava mass controls the rate of heat transfer. For smaller volcanoes a larger surface to volume ratio leads to faster relative cooling due to radiation from the free surface and conduction to the basal rock.

• As the lava viscosity increases due to cooling, the lava experiences higher drag from the ground. This in turn slows down the lava as it flows through the mountainous terrain and limits its run-out distance. The higher viscosity leads to more centralised, thicker and rounded lava flow on the valleys and a semi-circular pattern as it flows on the sea surface.

Larger volcanoes will therefore have much higher destructive power not only due to their sheer size but also because the lava flowing from them takes much longer to cool down and is thus significantly more free flowing, has more momentum and travels much further.
Identical eruptions with constant and a temperature dependent viscosity of lava led to important differences:

- The lava filling the valleys is more rounded, centrally flowing and thicker for the case with solidification. The lava deposits in the valleys are much wider for the constant viscosity case since the low viscosity lava is free to flow sideways. This is the effect of omitting solidification on smaller scale volcanoes.
- The lava front progresses almost linearly with time for the constant viscosity case whereas the flow stops its outward progression once the lava becomes sufficiently viscous when the viscosity is temperature dependent.
- The lava spreads out in a wedge like pattern as it flows into the sea for the constant viscosity case as opposed to a semi-circular flow pattern observed with lava with a temperature dependent viscosity.
- With a temperature dependent viscosity the lava piles-up into multiple layers as it enters the sea surface due to cooling and subsequent solidification. No such piling up effect is seen with a constant viscosity model.

SPH therefore offers several advantages over traditional CFD methods for volcanic lava flow simulations including:

- Full three dimensional simulations using real topography.
- Solution of the full Navier-Stokes equations coupled to heat transfer and solidification.
- Easy implementation of complex viscosity versus temperature relationships, and
- Prediction of pile-up of the lava as it becomes more viscous and solidifies. This in turn will be important for accurately predicting run-out distances.
References


Figure Captions

Figure 1: Digital Terrain Model (DTM) of a Californian Range obtained from the US geological survey (USGS) website.

Figure 2: Schematic of the simulation setup for generating lava flow from a conduit.

Figure 3: Steps involved in the simulation of a volcanic lava flow from set-up to visualisation.

Figure 4: Aerial view of the lava flow from a volcano with a conduit diameter of 2.9 m using the coupled thermal/solidification/fluid flow model. Red indicates high temperature and low viscosity and black indicates a low temperature and high viscosity.

Figure 5: Front view of the lava flow from a volcano with a conduit diameter of 2.9 m using the coupled thermal/solidification/fluid flow model. Red indicates high temperature and low viscosity and black indicates a lower temperature and high viscosity.

Figure 6: Close-up of lava flow showing pile-up of lava as it flows onto the plane representing the sea and solidifies due to cooling.

Figure 7: Comparison between lava flows for three different conduit sizes; (left) 29.0 m, (middle) 8.7 m and (right) 2.9 m.

Figure 8: Comparison of run-out distances for eruptions with conduit diameters of 2.9, 8.7 and 29.0 m. The distances are normalised to smallest eruption size. The inflow velocity is scaled for each simulation. The increasing importance of the cooling and solidification can be seen by the retarded run-out of the medium scale lava flow and the asymptotic behaviour of the small scale case.

Figure 9: Comparison between lava flow for constant (left column) and variable viscosity (right column) cases for a small scale eruption from a volcano with a conduit diameter of 2.9 m.

Figure 10: Comparison between run-out distances predicted for constant and variable viscosity lava flows, for the small eruption with a conduit diameter of 2.9 m.
Table Captions

Table 1: Exit velocities of lava erupting from the crater calculated on the basis of the conduit diameter of the volcano.
Figure 1
Volcanic crater
Conduit
Lava source

0.3 to 3.0 m
2.9 to 29.0 m

Figure 2
Figure 3
Figure 4
Figure 5
Figure 7
Figure 8
Figure 10