The influence of particle shape on flow modes in pneumatic conveying

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Abstract

The transportation of particles along pipes or ducts using an imposed gas flow is known as pneumatic conveying. The type of granular flow in such systems is strongly dependent on the imposed gas flow rate, and can be categorised by a distinct set of modes. These modes range from dilute flow, where the grains are entirely suspended in the gas, to moving dunes and slug flow, in which the bore of the pipe is blocked by a slow moving plug of material. Understanding the transitions between these modes is critical to the design and application of pneumatic conveying systems. Particle shape is a crucial factor in systems with gas-grain interactions but has so far been overlooked in models of pneumatic conveying. We carry out a series of simulations using the Discrete Element Method coupled to gas flow and show that particle shape is critical to the transition between different flow modes. Particles which are spherical, or nearly spherical, transition to slug flow at high gas flow rates, whereas non-spherical particles transition instead to dilute flow. We show the lower voidage fraction in beds of non-spherical particles.

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particles is crucial to explaining this behaviour.

Key words: Pneumatic conveying, Mathematical modelling, Simulation, Fluid mechanics, Discrete Element Model;

1. Introduction

A popular method for the rapid transportation of large volumes of granular material is pneumatic conveying, in which particles are driven along pipes using an imposed gas flow. The bulk grain motion can occur in several flow modes depending on parameters such as grain inflow rate, imposed gas flow rate, grain density and diameter (Molerus, 1996). The motion is usually classified into two phases; dilute phase, where the grains move as a suspension in the conveying gas, and dense phase, where the grains are subject to long lasting collisions and move in bulk along the pipe (Jones and Williams, 2008). One of the earliest investigations of this behaviour was carried out by Wen and Simons (1959), who used a glass pipe to observe pneumatic conveying, and sketched the various flow modes. They noted that the dilute flow phase occurred during high velocity gas flow, and at lower gas velocities the bed moved as a sequence of dunes. Dixon (1979) investigated the flow mode transitions for various particle categories suggested by Geldart (1973), specifically relating dense phase flow to the various schemes and produced a ‘slugging diagram’ for predicting the flow mode. This was later expanded by Molerus (1981) into state diagrams for both dilute and dense phase flow. Laouar and Molodtsof (1998) carried out a series of experiments to measure the pressure drop in dense phase conveying and noted the transition between various flow modes. Pan (1999) introduced the use of the ‘loose-poured’, or
bulk, density rather than the particle density to give a more accurate prediction of the flow mode. A summary of flow mode prediction using these approaches is given by Jones and Williams (2008).

Wen and Simons (1959) noted that ‘depending on the nature of solids and gas-solids ratio, intermittent flow of gas and solids in alternate slugs will occur, rather than dune formation’. In this mode, known as slug flow, particles initially sediment in a layer over the bottom of the pipe. The particle bed then rises to fill the bore of the pipe and the pressure builds up behind this blockage, pushing a slug out from the front of the particle bed. If the section of pipe in front of the slug is empty, the slug collapses into a layer of particles and the sequence repeats. A steady sequence of slugs only occurs if particles have already been deposited over the length of the pipe (Tomita et al., 1981). Due to low power consumption and low particle attrition (Pan and Wypych, 1997), this slug flow mode has received a large amount of attention both experimentally and in computational modelling. The presence of slugs can also be experimentally inferred from visual inspection of the flow using a transparent tube (Woods et al., 2008; Tomita et al., 2008). Slug flow can also be detected using a variety of other approaches, such as pressure measurements from a series of pressure transducers attached along the transporting pipe (Li et al., 2002). Takei et al. (2004) used capacitance computed tomography in which radial electrodes measure the electric field around a pipe during conveying. By measuring the capacitance between each electrode the permittivity, and hence particle concentration, can be determined.

Computationally, slug flow formation and collapse was first modelled by
Tsuji et al. (1992), who found good agreement with the predicted and measured stationary layer as well as overall slug dynamics. Fraigne and Langston (2006) modelled pneumatic conveying and reported good comparison to experimental observations. However, their model used a steady state assumption for the gas flow field, and based the pressure drop on the Ergun relation rather than any computed flow field. Tsuji et al. (2007) later extended his original work to a multiscale approach and demonstrated applicability in a number of areas including slug flow in pneumatic conveying. Strauß et al. (2007) compared horizontal slug formation to vertical plug formation, and investigated shear layering within the slug as well as slug collapse. Kuang et al. (2008) carried out detailed measurements of pressure drops as well as internal force networks within slugs and proposed an empirical relation between voidage fraction and particle flow rate over different gas flow rates. Sakai and Koshizuka (2009) have recently demonstrated slug flow in a nominally multiscale discrete element model.

The effect of shape on pneumatic conveying has not previously been investigated. A related problem of ellipsoidal particles in channel flow has been considered by Mortensen et al. (2008), although not in the context of pneumatic conveying. Particles in processing applications are rarely spherical and shape is known to strongly influence granular flow (Langston et al., 1994; Vu-Quoc et al., 2000; Cleary, 2008; Fraigne et al., 2008). The effect of particle shape has been widely investigated for a range of industrial applications and it has been shown that granular flow in mixing, hopper flow and comminution differ for spherical and non-spherical particles. The use of shaped particles in computational models of mixers have been shown to give
more realistic mixing rates than a spherical model (Cleary et al., 1998; Cleary and Metcalfe, 2002). Shape also strongly affects comminution processes in mills, where angular particles are found to cause a higher power draw than spherical particles (Cleary, 2001). In hoppers, particles with large aspect ratios have reduced flow rates compared to spherical equivalents, although the angularity of the particles makes only a small difference to the granular flow (Cleary and Sawley, 2002). The effects of particle shape in many industrial applications are summarised in Cleary (2004, 2009). The transition between the various pneumatic conveying regimes for different shaped particles is the focus of this paper.

2. Formulation

We use the Discrete Element Method coupled to the Navier Stokes equations for gas flow through a particle bed to computationally model the pneumatic conveying system. The gas flow through the bed has an interstitial velocity, \( \mathbf{u} \), which is the gas velocity through the pores of the bed, and a superficial velocity, \( \mathbf{u}' \), which is the average local velocity over some region of the bed. The two velocities are related by \( \mathbf{u}' = \epsilon \mathbf{u} \), where \( \epsilon \) is the bed voidage fraction. Each particle with index \( i \) has a velocity \( \mathbf{v}_i \), and the averaged particle velocity over a local region within the bed is given by \( \mathbf{v} \).

In the following sections we will give details of each element of the computational model.

2.1. Drag force

The drag force on the particles is the dominant gas-particle interaction in the system. The drag force exerted on a isolated particle, \( \mathbf{F}_{D0} \), subject to
a relative gas velocity $\mathbf{u}_r$ is defined by:

$$F_{D0} = \frac{1}{2} \rho_g |\mathbf{u}_r|^2 C_D A_\perp \mathbf{u}_r$$  \hspace{1cm} (1)$$

where $C_D$ is the gas drag coefficient, $\rho_g$ is the gas density and $A_\perp$ is the particle cross-sectional area perpendicular to the flow.

In systems composed of many particles the velocity is relative to the gas flow within the porous media, so $\mathbf{u}_r = \mathbf{u} - \mathbf{v}$. This individual drag force must also be modified to account for the flow and pressure effects from surrounding particles. Di Felice (1994) proposed the following voidage function correlation for the drag force, $F_D$, using an empirical fit to experimental data:

$$F_D = F_{D0} \epsilon^{-\chi}$$  \hspace{1cm} (2)$$

where $\chi$ is given by:

$$\chi = 3.7 - 0.65 \exp \left[ -\frac{1}{2} (1.5 - \log Re_p)^2 \right]$$  \hspace{1cm} (3)$$

The particle Reynolds number, $Re_p$, for a particle of radius $r$ in a gas with viscosity $\eta$ is $Re_p = 2r \rho_g |\mathbf{u}_r|/\eta$. The parameter $\chi$ is calculated for non-spherical particles by using the diameter of a sphere with equivalent volume to the non-spherical particle.

The drag coefficient must be modified if the particles are non-spherical. Hölder and Sommerfeld (2008) give the following drag coefficient for isolated non-spherical particles, based on correlations to experimental data given by Leith (1987), Ganser (1993) and Tran-Cong et al. (2004):
\[ C_D = \frac{8}{Re_p \sqrt{\Phi}} + \frac{16}{Re_p \sqrt{\Phi}} + \frac{3}{\sqrt{Re_p \Phi^2}} + 0.42 \times 10^{0.4(-\log \Phi)^{0.2}} \frac{1}{\Phi_{\perp}} \]  

(4)

where \( \Phi \) and \( \Phi_{\perp} \) are two sphericity measures for the non-spherical particle. The regular sphericity, \( \Phi \), is the ratio between the surface area of the volume equivalent sphere and the surface area of the considered particle. The cross-wise sphericity, \( \Phi_{\perp} \), is the ratio between the constant cross-sectional area of the volume equivalent sphere, \( A \), and the projected cross-sectional area of the considered particle perpendicular to the flow, \( A_{\perp} \), giving \( \Phi_{\perp} = A/A_{\perp} \). Eq. (4) is valid over the full practical range of \( Re_p \) (Hölzer and Sommerfeld, 2008). We use Eq. (4) to calculate the drag coefficient for both spherical and non-spherical particles in our simulations.

A Stokesian rotation drag term, \( T_D \), is also included in our implementation. This term incorporates the rotational drag on a spinning particle from the surrounding gas into our model. The drag is given by:

\[ T_D = 8 \pi \eta r^3 \omega_r \]  

(5)

where the relative particle spin is given by \( \omega_r = \frac{1}{2} \omega_g - \omega_p \), with \( \omega_g \) the curl of the gas velocity field and \( \omega_p \) the spin of the particle.

2.2. Lift forces

The lift forces play an important role in the development of bed dynamics for pneumatic conveying and other gas particulate transport problems such as dune formation. There are several components to the lift force on a particle. These are the shear-dependent lift force (Saffman force), the rotational-dependent lift force (Magnus force) and the acceleration-dependent lift force.
(Basset force). The Basset force is negligible in gases at low particle acceleration so the lift, $\mathbf{F}_L$, is taken to be the sum of the Saffman force, $\mathbf{F}_{LS}$, and the Magnus force, $\mathbf{F}_{LM}$. The Saffman lift on an isolated particle is given by a generalisation of Saffman’s one-dimensional form (Saffman, 1964), given by Drew (1976):

$$\mathbf{F}_{LS} = 6.46C_{LS}r^2\sqrt{\rho_g\eta} \frac{\mathbf{u}_r \cdot \mathbf{D}}{\sqrt{\mathbf{D}}}$$  \hspace{1cm} (6)

where the rate of deformation tensor is given by $\mathbf{D} = \nabla \mathbf{u}_r + \nabla \mathbf{u}_r^T$. An empirically corrected Saffman lift coefficient, $C_{LS}$, is given by Mei (1992):

$$C_{LS} = \begin{cases} e^{-0.1Re_p} - 0.3314\sqrt{\alpha}(e^{-0.1Re_p} + 1) & Re_p \leq 40 \\ 0.0524\sqrt{\alpha Re_p} & Re_p > 40 \end{cases}$$

where the coefficient $\alpha = |\mathbf{D}|r/|\mathbf{u}_r|$.  

The Magnus lift force is given by:

$$\mathbf{F}_{LM} = \frac{1}{2}C_{LM}\pi r^2 \rho_g \frac{\mathbf{\omega}_r \times \mathbf{u}_r}{|\mathbf{\omega}_r|}$$  \hspace{1cm} (7)

where we use a Magnus lift coefficient, $C_{LM}$, is given by Lun and Liu (1997):

$$C_{LM} = 2r|\mathbf{\omega}_r| \begin{cases} 1 & Re_p \leq 1 \\ (0.178 + 0.822Re_p^{-0.522}) & Re_p > 1 \end{cases}$$

No straightforward orientation dependent terms, such as $A_\perp$, are contained in these expressions and there is no clear approach to modifying the expressions to account for particle shape. Detailed experimental measurements are required to develop correlations that account for particle shape.
in these lift forces. Furthermore, no empirical correction to the lift forces accounting for the presence of nearby particles could be found. Due to these factors the lift terms for non-spherical particles were implemented by assuming these were isolated spheres with equivalent volumes to that of the non-spherical particles.

2.3. Particle dynamics

The force balance on each particle $i$ is given by:

$$m_i \frac{dv_i}{dt} = F_{Ci} + F_{GPi} + m_i g \tag{8}$$

where $m_i$ is the mass of particle $i$, and $v_i$ is the velocity. The forces consist of a solid particle-particle contact force, $F_C$, a gas-particle interaction force, $F_{GP}$, and the gravitational force $mg$. The gas-particle interaction force, $F_{GP}$, is the sum of the gradient of the gas pressure, $p$, the particle drag force, $F_D$, the Saffman lift force, $F_{LS}$, and the Magnus lift force, $F_{LM}$:

$$F_{GPi} = -V_i \nabla p + F_{Di} + F_{LSi} + F_{LMi} \tag{9}$$

The moment balance on each particle is given by:

$$I_i \frac{d\omega_i}{dt} = T_i + T_{Di} \tag{10}$$

where $I_i$ is the moment of inertia in the principal frame of the particle, $T_i$ is the inter-particle contact torque, $T_{Di}$ is the Stokesian rotational drag on the particle and $\omega_i$ is the spin of the particle.

The computation of these forces for a large set of individual particles forms the basis of the Discrete Element Method (DEM), first formulated
by Cundall and Strack (1979). In this implementation, the particle-particle contact force, $F_{ci}$, is determined by the particle overlap information using a linear spring, dashpot and slider approximation. This force is the sum of a normal force $F_n$ and a tangential force $F_t$ at the contact point. The normal force is determined from the particle overlap $\delta l$ and relative normal velocity $v_n$:

$$F_n = -k_n \delta l + C_n v_n$$  \hspace{1cm} (11)

where $C_n$ is the normal damping coefficient, chosen to give the required normal coefficient of restitution, and $k_n$ is the spring stiffness. The spring stiffness determines the maximum particle overlap, which should be around $\sim 0.1\%$ to give an accurate simulation within a reasonable computational timeframe.

The tangential force is given by:

$$F_t = \min \left[ \mu F_n, k_t \int v_t \, dt + C_t v_t \right]$$  \hspace{1cm} (12)

where $v_t$ is the relative tangential velocity, $\mu$ is the coefficient of friction between the particles and the walls, $C_t$ is the tangential damping coefficient and $k_t$ is the tangential spring stiffness. The integral term models the tangential elastic deformation of the surface and the force is limited by the Coulomb friction limit $\mu F_n$. When this limit is overcome, the surfaces in contact slide over each other.

Several approaches can be used to model non-spherical particles. These include modelling the particles as ellipses (Rothenburg and Bathurst, 1991), polygonal shapes (Walton, 1983, 1984; Hopkins et al., 1991) and clusters of
spherical particles (Jensen et al., 1999). Here, super-quadrics are chosen to simulate non-spherical particles as they give an excellent trade-off between speed and shape flexibility. The super-quadric is defined by the closed surface given by:

\[
\left(\frac{x}{a_x}\right)^q + \left(\frac{y}{a_y}\right)^q + \left(\frac{z}{a_z}\right)^q - 1 = 0
\] (13)

where \(a_x\), \(a_y\) and \(a_z\) is the aspect ratios in the \(x\), \(y\) and \(z\) directions in the canonical frame of the particle, respectively, and the index \(q\) indicates the blockiness of the particle. Ellipsoids have \(q = 2\) and higher values of \(q\) give increasingly blocky particles, becoming cubical in the limit \(q \to \infty\). Representative particles of various shape are shown in Fig. 1.

Contact detection between the super-quadric particles is carried out to find the particle overlap. This is achieved by numerically calculating the closest distance between the two overlapping super-quadric surfaces. This overlap length is then used in the DEM model. This use of super-quadrics was first suggested by Williams and Pentland (1992) in 2D for a much more general context. The first 3D application of super-quadrics to DEM was carried out by Cleary (2004) and has since been applied in a broad range of engineering applications (Cleary, 2009). Numerical stability is achieved by ensuring each collision is properly resolved by taking an appropriate number of time steps during each collision.

The drag expression contains an orientation-dependent term, \(A_\perp\), which has previously been found to be critical to the fluidisation behaviour of a particle bed of shaped particles (Hilton et al., 2010). We numerically calculate this flow projected area from the projected rim of the super-quadric
onto a plane orthogonal to the relative local flow direction. This ensures an accurate measurement of the projected face area which is crucial to predicting the flow dynamics. Prolate elliptical particles with their long axis orientated into the flow have reduced drag compared to an equivalent sphere while oblate ellipsoids with their short axis orientated into the flow have, correspondingly, an increased drag. The inclusion of the relative rotational orientation of the particles using this method ensures the flow dynamics of non-spherical particles are accurately modelled.

2.4. Fluid dynamics

We use a reformulation of the constitutive equations for gas flow through a particle bed for modelling the fluid dynamics in the system (Hilton et al., 2010). The original form of the constitutive equations was derived by Anderson and Jackson (1967), and are given in a ‘pressure gradient force’ (PGF) form by Kafui et al. (2002) as:

\[
\frac{\partial (\epsilon \rho_g)}{\partial t} + \nabla \cdot (\epsilon \rho_g \mathbf{u}) = 0 \tag{14}
\]

\[
\frac{\partial (\epsilon \rho_g \mathbf{u})}{\partial t} + \nabla \cdot (\epsilon \rho_g \mathbf{u} \mathbf{u}) = -\epsilon \nabla p - f_{sp} - \nabla \cdot (\epsilon \mathbf{\tau}) + \epsilon \rho_g \mathbf{g} \tag{15}
\]

where \( \epsilon \) is the local bed voidage fraction, \( \rho_g \) the gas phase density, \( \mathbf{u} \) the interstitial velocity field, \( p \) the pressure, \( \mathbf{g} \) the gravity vector and \( \mathbf{\tau} \) the local stress tensor given by:

\[
\mathbf{\tau} = -\eta \left[ \nabla \mathbf{u} + \nabla \mathbf{u}^T - \left( \frac{2}{3} \nabla \cdot \mathbf{u} \right) \delta \right] \tag{16}
\]
where $\delta$ is the unit tensor. The particle-gas interaction body force, $f_{gp}$, is defined as:

$$f_{gp} = \frac{1}{V_c} \sum_{i=1}^{n_c} F_{Di} = \frac{1}{V_c} \sum_{i=1}^{n_c} \frac{1}{2} \rho_g |\mathbf{u} - \mathbf{v}_i| C_D A \epsilon (2 - \chi)(\mathbf{u} - \mathbf{v}_i)$$

(17)

where $n_c$ is the number of particles in the characteristic gas volume $V_c$.

These expressions can be re-formulated into relations for superficial gas velocity, $u'$, rather than the interstitial gas velocity, $u$, by making the assumption that the gas density, $\rho_g$, is constant. This gives:

$$\frac{\partial \epsilon}{\partial t} = -\nabla \cdot \mathbf{u}'$$

(18)

$$\frac{\partial \mathbf{u}'}{\partial t} + \frac{1}{\epsilon} (\mathbf{u}' \cdot \nabla) \mathbf{u}' + \mathbf{u}' \nabla \cdot \left( \frac{1}{\epsilon} \mathbf{u}' \right) =$$

$$-\frac{1}{\rho_g} \left[ \epsilon \nabla p_f + f_{fp} + \nabla \cdot (\epsilon \mathbf{tau}) \right] + \epsilon \mathbf{g}$$

(19)

where the stress tensor $\mathbf{tau}'$ is given by:

$$\mathbf{tau}' = -\eta \left[ \left( \frac{\nabla \mathbf{u}'}{\epsilon} \right) + \left( \nabla \frac{\mathbf{u}'}{\epsilon} \right)^T - \left( \frac{2}{3} \nabla \cdot \frac{1}{\epsilon} \mathbf{u}' \right) \delta \right]$$

(20)

Eqs. (18) and (19) make up our constitutive model. Due to the moderate Reynolds numbers in the pneumatic conveying system considered here ($\sim 10^3$) no turbulence terms are included.

The gas flow is numerically calculated using a modification of the finite-difference pressure correction method, using Eq. (18) as a source term (Hilton et al., 2010). This method is more straightforward than methods based on
compressible flow equations, which require an extra expression for the energy term as well as an equation of state. The equations are discretised onto a semi-staggered Cartesian grid, to reduce checkerboard pressure oscillations, with velocity and force defined on the cell vertices and voidage fraction, pressure and the stress tensor defined at cell centres. Advection terms are calculated using upwind biased methods and divergence, curl and gradient terms are calculated using semi-staggered finite difference stencils. The forces on the gas from the particles are mapped using collocation, whereas the forces from the gas on the particles are mapped using tri-linear interpolation. Time integration is carried out using a second-order Runge-Kutta method.

The fluid time step is governed using the discrete element time step. However, if this required time step is larger than the maximum stable time step the fluid solution can take, the fluid solver takes multiple sub-steps. This is rarely the case, however, and both solutions normally run with the same time step. The maximum stable time step is calculated using a fraction (0.1) of the maximum allowable time step from both the Courant - Friedrichs - Lewy condition and a force time step condition. This ensures that the linking between the particles and fluid is numerically stable. The fluid solution is carried out within the discrete element time step. The fluid solution uses the positions and velocities of the particles at the start of the discrete element time step to calculate the fluid - particle interaction forces. These forces are applied in the discrete element solver after the fluid time step has finished.

The voidage fraction within each grid cell is calculated from the fraction of each super-quadric particle overlapping the cell using a recursive octree method, given in Hilton et al. (2010). This method gives a highly accurate
measure of the voidage fraction within each computational cell, which is critical for modelling the correct dynamics of the system.

2.5. Boundary conditions

We use a rectangular domain as shown in Fig. 2. Periodic boundary conditions are applied for the gas and particles in the stream-wise $x$ direction, allowing the particles and gas to re-circulate until steady state is reached. To drive the gas we use a pressure gradient imposed over the gas. The system pressure is split into a fluctuating and steady part, $p = p_f + p_s$, such that the gradient of $p_s$ gives a driving pressure gradient. From Eq. (19) this gives:

$$\frac{\partial u'}{\partial t} + \frac{1}{\epsilon} (u' \cdot \nabla)u' + u' \nabla \cdot \left( \frac{1}{\epsilon} u' \right) =$$

$$-\frac{1}{\rho_g} \left[ \epsilon \nabla p_f + f_f + \nabla \cdot (\epsilon \tau) \right] + \epsilon \left( g - \frac{\nabla p_s}{\rho_g} \right) \quad (21)$$

The pressure gradient can therefore simply be imposed as a body force, along with gravity. The term $\nabla p_s/\rho_g$ has units of acceleration, and can be converted to an equivalent pressure gradient by multiplying by the gas density. The imposed pressure gradient can be easily controlled in this way, and is transparent to periodic boundary conditions. The remaining boundary conditions are a solid no-slip boundary in the upper and lower $y$ directions for both particles and gas. Periodic boundary conditions are applied in the transverse $z$ direction for both particles and gas, to remove any lateral boundary effects.
3. Pneumatic Conveying Configuration

The set-up is shown in Fig. 2, consisting of a rectangular duct of length 10 cm with width and height of 1 cm. The lower half of the duct is evenly filled with particles of varying shape. Our parallel plate set-up represents a central slice from a much wider rectangular duct. This is different to cylindrical ducts which have radial compression effects. The geometry chosen allows us to understand the gas-particle coupling in the simpler rectangular set-up before considering more complex conveying systems with other duct cross-sections.

We use a range of ellipsoids from oblate through spherical to prolate. A set of simulations with cuboidal particles was also carried out to investigate the flow modes for angular particles. The shape parameters of the particles used are given in Table 1, and the computational parameters used are given in Table 2. We consider particle sizes of 400 µm equivalent spherical radius for the ellipsoidal particles and 300 µm for the cuboidal particles. The particles were approximately monodisperse, with a 1% variation in radius to prevent crystallisation effects. The density was 1000 kg/m³, on the border between Geldart groups B and D (Geldart, 1973). The particles were assumed to be cohesionless, with an inter-particle friction of µ = 0.5 and coefficient of restitution 0.5. The particles were driven by an imposed gas flow along the length of the domain. The particle diameter to grid size ratio was 2 : 1, giving an optimal grid resolution for the method. The normal spring stiffness was $10^3$ N/m to give a maximum particle overlap of $\sim 0.1\%$.

The pressure gradient chosen ranged from 0.6 kPa/m to 6.0 kPa/m in non-uniform increasing increments in order to capture the behaviour of the
bed over a wide range of driving conditions. This range was chosen to match pressure gradients around the values measured by Laouar and Molodtsof (1998), which ranged up to a maximum of $\sim 10.0 \, kPa/m$ in their experiments. To ensure that periodicity did not affect the outcome of the simulations, several set-ups were repeated in domains of length 13 cm and 20 cm. These gave very similar results to the 10 cm domain.

4. Flow Transitions

The behaviour of the system was investigated in a series of simulations at fixed pressure gradients. Cross sections of the particle bed with particles shaded by velocity magnitude are shown for ellipticity 1.0 (spherical) particles in Fig. 3 and ellipticity 0.5 (prolate ellipsoid) particles in Fig. 4.

The bed is stationary for pressure gradients giving a gas flow under the particle saltation velocity, which is $\sim 0.25 m/s$ using an expression given by Dallavalle (1948). For gas flow velocities above the saltation velocity, surface particles lift off from the bed and are carried along in suspension by the gas. At low pressure gradients the bed is stationary and the steady state flow is a dilute suspended stream of saltating particles moving over a fixed bed, as shown in Fig. 3 at a pressure gradient of 0.6 kPa/m to 1.2 kPa/m and Fig. 4 at 0.6 kPa/m to 1.8 kPa/m. We label this flow mode as type ‘A’.

At higher pressure gradients the bed begins to shear and the steady state flow is a sequence of small dunes rolling over the particle bed under a dilute stream of particles. If the pressure gradient is increased, these dunes become larger and the shear layer extends to the bottom of the bed, resulting in net bed motion. This is shown in Fig. 3 at a pressure gradient of 1.8 kPa/m...
and Fig. 4 at 2.4 kPa/m to 3.0 kPa/m. This flow mode is labeled type ‘B’.

For higher pressure gradients the behaviour of the system is determined by the particle shape. In systems with elliptical or cuboidal particles the upper stream of suspended particles grows to fill the entire duct. This results in fully dilute particle flow, as shown in Fig. 4 at a pressure gradient of 6.0 kPa/m. This flow mode is labeled type ‘C’.

Systems with rounder particles (ellipticity 1.0 and 1.1) experience growth in the dunes until the duct is blocked. The blockage forms a slug which travels at a constant velocity down the duct, as shown in Fig. 3 at a pressure gradient of 2.4 kPa/m to 6.0 kPa/m. The slugs have a distinctive profile with a steeply sloped upstream head, in which particles are scooped up and overturn, and a long shallow sloped downstream tail, in which particles are deposited after the passage of the slug. The slugs move at a constant velocity and the gas velocity is reduced in the duct, compared to the other flow modes. We label the slug flow as type ‘D’.

Flow modes ‘A’ to ‘D’ in the simulations were the same as flow modes observed experimentally by Wen and Simons (1959) and Laouar and Molodtsof (1998), amongst others. Our categorisation is based on the modes given in Laouar and Molodtsof (1998), along with an additional mode ‘A’.

Fig. 5 shows a cross-section of the bed for particles with ellipticity 0.5, 1.0 and 1.5 as well as cuboidal particles, at steady state with a pressure gradient of 3.0 kPa/m. The spherical particles have formed a slug (mode D) but the ellipsoids and the cuboidal particles have a dilute stream of particles over a sheared bed (mode B).

A phase diagram for the system is shown in Fig. 6. It should be noted
that the pressure gradients applied to the system are not in linear steps, so as to capture the flow modes over a wide range of scales. Slug flow regime exists after the sheared bed mode (B) for particles with ellipticity 1.0 and 1.1. The other particles, with high and low aspect ratios, transition directly from the sheared bed mode (B) to the dilute flow mode (C).

Cuboidal particles showed very similar behaviour to high aspect ratio ellipsoidal particles (ellipticity 0.5 and 1.5). As the pressure gradient is increased cuboidal particles transition from fixed bed flow (A) through sheared bed flow (B) directly to dilute flow (C) with no slug formation. The sheared layer in flow type B was found to be slightly thinner for cuboidal particles than for high aspect ratio ellipsoidal particles at the same pressure gradient.

5. Slug Dynamics

A slug is a steady state blockage, moving at a constant velocity. Both the slug velocity and gas velocity were measured in the simulation with spherical particles with a pressure gradient of 2.4 kPa/m, after the slug had formed. The slug was measured to move at a constant velocity of 0.52 m/s. The average gas pressure along the bed at 0.6 s is shown in Fig. 7. The vertical dashed lines show the boundaries of the slug, from 0.5 cm to 3.0 cm. The average pressure gradient within the slug is −4.8 kPa/m, and the pressure gradient outside the slug is 2.4 kPa/m, which is exactly the pressure gradient imposed along the duct.

As the slug is in a steady state, the gas velocity field is also in a steady state. The superficial gas velocity within the slug was found to be $|u'| = 0.66$ m/s. The relative velocity of the gas inside the slug, $u_r$, is the superficial
velocity, $u'$ minus the slug velocity, $v_b$, $u_r = u' - v_b$, giving $|u_r| = 0.14 \text{ m/s}$ for this case. We can use this to compare our results to the empirical Ergun relation (Ergun, 1952):

$$\frac{\Delta p_{gas}}{l} = \frac{150\eta(1 - \epsilon)^2}{(2r)^2e^3} |u_r| + \frac{1.75\rho_g(1 - \epsilon)}{2re^3} |u_r|^2$$

(22)

where $\Delta p_{gas}$ is the net pressure gradient over the slug and $l$ is the length of the slug. The pressure gradient is $4.8 \text{ kPa/m} - 2.4 \text{ kPa/m} = 2.4 \text{ kPa/m}$, which gives an interstitial velocity from the Ergun relation of $0.13 \text{ m/s}$. This result is in good agreement with our simulated value of $0.14 \text{ m/s}$.

6. Comparison to Experimental Results

To validate our model we compare data from the steady state slug flow modes (type ‘D’) to experimental results from Wen and Simons (1959) and Woods et al. (2008). The data is non-dimensionalised in a form given by Wen and Simons (1959), which we label $K$:

$$K = \left( \frac{1}{\rho_b g \frac{\partial p}{\partial x}} \right) \left( \frac{D}{d} \right)^{\frac{1}{4}}$$

(23)

where $D$ is the pipe diameter or duct width and $d$ is the particle diameter. We also non-dimensionalise the data using the Froude number, $Fr$:

$$Fr = \frac{|v_{av}|}{\sqrt{gD}}$$

(24)

where $v_{av}$ is the average particle velocity in the system. This is calculated for our simulations as the average particle velocity within a $3 \text{ mm}$ wide slice perpendicular to the $x$ direction in the centre of the simulation domain,
temporally averaged over the entire duration of the simulation. The bulk density is measured within the slug, and $\frac{\partial p}{\partial x}$ is the imposed gas pressure gradient over the system.

Wen and Simons (1959) use a ‘dispersed solid density’ in their corresponding data plot, which is converted to a bulk density by dividing by their parameter $k$. This is taken to be 2.5, the average of their range of $k$, $(2 - 3)$. Woods et al. (2008) use a solids mass flow rate, $W$, which is converted to an average particle velocity using $|\mathbf{v}_{av}| = W/(A\rho_b)$, where $A$ is the cross sectional area of their pipe. The comparison is plotted logarithmically in Fig. 8, along with a fitting curve from Wen and Simons (1959). This fitting curve is given in S. I. units as:

$$K = 1.7|\mathbf{v}_{av}|^{0.45} \quad (25)$$

The fitting curve is calculated using our duct width, $D = 1 \text{ cm}$. Our simulation results show excellent agreement with the data provided by Wen and Simons (1959) and lie on their fitting curve. Data sets from Woods et al. do not overlap our ranges of $K$ and $Fr$, but most of the data lies on the same curve. The steel shot data appears to lie under the curve, and this over-prediction of the pressure by Wen and Simon’s relation is mentioned by Woods et al. Overall, however, the pressure drop and particle velocities in our dense phase systems closely match experimental data values.
7. Slug Stability

7.1. Wall Stress Model

We have shown that pneumatic conveying of particles in a rectangular duct only produces slug flow for particles which are spherical, or close to spherical. This behaviour can be explained using a variation on a force balance given by Mi and Wypych (1994), similar to an argument given by Dixon (1979) for plug behaviour in vertical tubes.

If a blockage of particles forms within the pipe, it will either be unstable and disperse, or be stable and form a steady state slug. The stability of the blockage is determined by two opposing body forces. There is a body force from the gas-grain interaction driving the blockage down the pipe, provided by a gas pressure gradient \( \frac{\partial p}{\partial x} \), which acts in the direction of the gas flow. This is opposed by a body force from the frictional resistance of the blockage to the walls of the pipe, as well as internal friction from the particles sliding over each other, \( \frac{\partial p}{\partial x} \).

The driving gas pressure gradient over the blockage can be approximated by the Ergun relation, Eq. 22, as the blockage is in a partially aerated state. The gas pressure gradient can be assumed to be constant over the blockage \( \frac{\partial p}{\partial x} \) = \( \Delta p_{\text{gas}} / l \).

The opposing frictional force can be determined using an approach derived for a cylindrical tube given by Mi and Wypych (1994). This is reformulated here for a rectangular duct. The frictional pressure gradient within the blockage, \( \frac{\partial p}{\partial x} \), is composed of a resisting pressure resulting from the grain-wall interaction, \( \partial \sigma_{\text{wall}} / \partial x \), as well as the inter-grain interaction and resistance of the stationary layer of the bed, \( \partial \sigma_{\text{bed}} / \partial x \).
\[
\left[ \frac{\partial p}{\partial x} \right]_{fr} = \frac{\partial \sigma_{wall}}{\partial x} + \frac{\partial \sigma_{bed}}{\partial x}
\]

(26)

It is assumed that the wall and bed resistance are much greater than inter-grain forces, which are neglected. From the cohesionless Coulomb stress criterion the gradient of the wall stress is given by \( \partial \sigma_{wall} / \partial x = \mu \rho_b g \), where \( \rho_b \) is the bulk density of the particle bed, \( \rho_b = (1 - \epsilon) \rho_p \), and \( \mu \) is the wall friction coefficient. We assume any compressional stress of the blockage is small compared with the weight of the blockage and does not contribute to \( \partial \sigma_{wall} / \partial x \). We also assume that the frictional pressure gradient over the blockage is constant, \( \partial p / \partial x \) \( \left[ \frac{\partial p}{\partial x} \right]_{fr} = \Delta p_{fr} / l \). This gives:

\[
\frac{\partial \sigma_{bed}}{\partial x} = \frac{\Delta p_{fr}}{l} - \mu \rho_b g
\]

(27)

Integrating the above expression from \( x = 0 \) to \( x = l \) and applying the boundary conditions \( \sigma_{bed} = \sigma_{front}, x = l \), where \( \sigma_{front} \) is the stress at the front of the blockage, and \( \sigma_{bed} = 0, x = 0 \) gives:

\[
\frac{\Delta p_{fr}}{l} = \frac{\sigma_{front}}{l} + \mu \rho_b g
\]

(28)

Mi and Wypych (1994) give the following expression for \( \sigma_{front} \), based on a momentum balance:

\[
\sigma_{front} = \alpha \rho_b |v_b|^2
\]

(29)

where \( \alpha \) is ratio of stationary bed area to the cross sectional duct area and \( |v_b| \) is the speed of the blockage, or the speed of the slug if the blockage has
formed a steady state slug. Substituting Eq. 29 into Eq. 28 and re-arranging for the pressure gradient gives:

$$\frac{\Delta p_{fr}}{l} = \frac{\alpha \rho_b |v_b|^2}{l} + \mu \rho_b g$$

(30)

The two expressions for opposing pressure gradients, $\Delta p_{gas}/l$ from Eq. 22 and $\Delta p_{fr}/l$ from Eq. 30, can now be used to derive an expression for the stability of the blockage.

For a stable slug to form the driving gas pressure gradient must not exceed the bed resistance pressure gradient; if the blockage is driven harder than the bed resistance it will disintegrate. If, however, the resistance is greater than the driving pressure gradient the blockage will grow until the body forces become equal. The blockage will then move at a constant velocity, becoming a slug. To determine whether a blockage will become a stable slug we can therefore simply divide Eq. 30 by Eq. 22 to form a dimensionless stability parameter, $\gamma = \frac{\Delta p_{fr}}{\Delta p_{gas}}$. If this is greater than one, a blockage will form a stable slug. If it is less than one, any blockage will disintegrate and no slugs will form.

For the spherical case with a driving pressure gradient of $2.4 \text{ kPa/m}$ the slip velocity was measured from the simulations as $|u_r| = 0.14 \text{ m/s}$. The slug length is $l = 2.5 \text{ cm}$ and, from visual inspection of the particle bed, the bed fraction ratio $\alpha \sim \frac{1}{4}$. This gives a value of $\gamma = 1.26$, predicting the slug is stable, as observed in our simulation.

7.2. Non-Spherical Particles

If a blockage of the same length, $l$, occurs in a system with non-spherical particles under the same driving gas pressure gradient, we can construct a
simple model for evaluating whether this blockage will form a stable slug. The stress at the front of the blockage is dependent on particles pushing through the bed and governed by the particle mass and interaction with neighboring grains. All the particle types considered here have the same mass and all are convex, sharing only one point of contact with a neighboring particle. We therefore assume the stress at the front of the blockage, $\sigma_{\text{front}}$, is the same as the spherical case.

With this assumption, the free variables in the stability parameter, $\gamma$, are the voidage fraction, $\epsilon$ and the slip velocity $u_r$. The voidage fraction, $\epsilon$, was measured from the packed bed of each type of shaped particle used in the simulation and is given in Table 4. The spherical particles have the highest voidage fraction, which is a well-known result from static packing problems (Delaney and Cleary, 2009). Fig. 9 shows a stability diagram of $\gamma$ as a function of ellipticity and slip velocity using the measured values of $\epsilon$. The shaded region shows $\gamma > 1$, so if the slip velocity and ellipticity both fall within this region, stable slugs will form in the system. The slug of spherical particles with ellipticity 1 and $|u_r| = 0.14 \text{ m/s}$ is marked on this figure within the stable region.

This stability diagram shows that if $|u_r| > 0.16 \text{ m/s}$, slugs will only form in systems of spherical particles. Systems with shaped particles of ellipticity 0.9 and 1.1 will only form stable slugs if $|u_r| < 0.16 \text{ m/s}$. This is clearly consistent with our results, as it is likely that the slip velocity in blockages of shaped particles is similar, or slightly higher, than the slip velocity of $|u_r| = 0.14 \text{ m/s}$ measured for spherical particles. The model gives the same trend in stability observed in our simulations, and shows that small variations
in the voidage fraction can alter the overall dynamic balance between the driving pressure gradient and the bed resistance, leading to changes in slug stability.

8. Conclusion

Coupled DEM-gas flow simulations have shown that the shape of a particle can significantly affect the bulk dynamics of a pneumatic conveying system. The steady state flow at different pressure gradients was found to lie in one of four flow modes, summarised in Table 3. At low pressure gradients the flow consists of a dilute stream of suspended grains over a stationary bed of particles (type ‘A’). At higher pressure gradients this bed begins to shear and small dunes begin to form (type ‘B’). An increase in pressure gradient causes these dunes to grow until the duct is filled. Finally, a very high pressure gradient either causes a transition to fully dilute granular flow (type ‘C’), if the driving pressure gradient is higher than the bed resistance, or slug flow (type ‘D’), if the driving pressure gradient is lower than the bed resistance.

The transition to either type ‘C’ or ‘D’ flow modes can be determined using a simple force balance between the driving pressure gradient (Eq. 22) and the bed resistance (Eq. 30). It may be expected that ellipsoidal particles with high ellipticity would interlock more readily, allowing easier slug formation, but we have demonstrated that this is not the case. Slugs were found to be stable only when the grains are spherical, or close to spherical. In beds with ellipsoidal grains, the lower voidage fraction causes an increase in the pressure gradient within the slug, destabilising slug formation.
Some care must be taken applying these findings in a more general context as granular materials used in industrial systems are typically polydisperse and irregularly shaped. Systems such as these must be examined before more general conclusions can be drawn. However, shape has clearly been shown to play an important role in slug formation. Shape-dependent parameters, such as bulk density and the average particle area incident to the flow, are therefore crucial in the consideration and design of such systems.

Notation

\begin{itemize}
  \item \( A \) characteristic particle area, \( m^2 \)
  \item \( A_\perp \) projected particle area, \( m^2 \)
  \item \( a_{x,y,z} \) super-quadric shape factors, \( m \)
  \item \( C_D \) drag coefficient
  \item \( C_{LM} \) Magnus lift coefficient
  \item \( C_{LS} \) Saffman lift coefficient
  \item \( C_{n,t} \) normal, tangential damping coefficients, \( kg/s \)
  \item \( d \) Particle diameter, \( m \)
  \item \( \mathbf{D} \) Rate of strain tensor, \( 1/s \)
  \item \( D \) Pipe diameter or duct width, \( m \)
  \item \( \mathbf{F}_C \) solid particle-particle contact force, \( N \)
  \item \( \mathbf{F}_D \) single particle drag force in multi-particle system, \( N \)
  \item \( \mathbf{F}_{D0} \) single isolated particle drag force, \( N \)
  \item \( \mathbf{F}_{GP} \) gas-particle interaction force, \( N \)
  \item \( \mathbf{F}_L \) particle lift force, \( N \)
  \item \( \mathbf{F}_{LM} \) particle Magnus (rotational) lift force, \( N \)
\end{itemize}
$F_{LS}$ particle Saffman (shear) lift force, N

$f_{gp}$ particle-gas interaction body force, N/m$^3$

$g$ gravitational acceleration, m/s$^2$

$I$ particle moment of inertia, kg m$^2$

$k_{n,t}$ normal, tangential spring stiffness coefficients, N/m

$l$ slug length, m

$m$ particle mass, kg

$n_c$ number of particles in characteristic gas volume

$N$ number of particles in system

$p$ gas pressure, Pa

$q$ super-quadric shape exponent

$r$ characteristic particle radius, m

$Re$ Reynolds number

$T$ torque, N m

$T_D$ drag torque, N m

$u$ interstitial gas velocity, m/s

$u'$ superficial gas velocity, m/s

$u_r$ relative gas velocity, m/s

$v$ particle velocity, m/s

$v_b$ blockage or slug velocity, m/s

$v_{av}$ average particle velocity in system, m/s

$V_p$ particle volume, m$^3$

$V_c$ characteristic gas volume, m$^3$

$W$ mass flow rate, kg/s

$\delta l$ particle overlap, m
Greek letters

\[\alpha\] ratio of stationary bed area to cross-sectional duct area
\[\delta\] unit tensor
\[\epsilon\] voidage fraction
\[\eta\] gas viscosity, kg/m s
\[\gamma\] slug stability coefficient
\[\mu\] coefficient of friction
\[\rho_g\] gas density, kg/m\(^3\)
\[\rho_p\] particle solid density, kg/m\(^3\)
\[\rho_b\] particle bulk density \((1 - \epsilon)\rho_p\), kg/m\(^3\)
\[\tau\] viscous stress tensor, Pa
\[\tau'\] viscous superficial stress tensor, Pa
\[\Phi\] sphericity
\[\Phi_\perp\] crosswise sphericity
\[\sigma\] slug stress, Pa
\[\sigma_{\text{wall}}\] slug transverse wall stress, Pa
\[\sigma_{\text{bed}}\] slug bed stress, Pa
\[\sigma_{\text{front}}\] stress at front of slug, Pa
\[\chi\] voidage function exponent
\[\omega\] angular velocity, rad/s
\[\omega_g\] gas vorticity, rad/s
\[\omega_r\] particle spin, rad/s
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Figure 1:

**Prolate Ellipsoid**
Ellipticity 0.5
\[ q = 2, \ a_x = 1.0, \ a_y = a_z = 0.5 \]

**Sphere**
Ellipticity 1.0
\[ q = 2, \ a_x = a_y = a_z = 1.0 \]

**Oblate Ellipsoid**
Ellipticity 1.5
\[ q = 2, \ a_x = a_z = 1.0, \ a_y = 0.5 \]

**Cuboid**
\[ q = 4, \ a_x = a_y = a_z = 1.0 \]
Figure 2:
Figure 3:

Flow Direction

- 0.6 kPa/m, 1.85 m/s
- 0.9 kPa/m, 1.76 m/s
- 1.2 kPa/m, 1.78 m/s
- 1.8 kPa/m, 1.96 m/s
- 2.4 kPa/m, 0.75 m/s
- 3.0 kPa/m, 0.87 m/s
- 6.0 kPa/m, 1.49 m/s
Figure 4:

Flow Direction

<table>
<thead>
<tr>
<th>Pressure (kPa/m)</th>
<th>Velocity (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>1.84</td>
</tr>
<tr>
<td>0.9</td>
<td>2.02</td>
</tr>
<tr>
<td>1.2</td>
<td>1.98</td>
</tr>
<tr>
<td>1.8</td>
<td>2.15</td>
</tr>
<tr>
<td>2.4</td>
<td>2.36</td>
</tr>
<tr>
<td>3.0</td>
<td>2.72</td>
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<tr>
<td>6.0</td>
<td>4.28</td>
</tr>
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</table>
Figure 6:
Figure 7:
Figure 8:

\[ \kappa = \left( \frac{1}{\rho g x} \right) \frac{D}{d} \]
Figure 9:

- **Unstable**
- **Unstable**
- **Stable**

Stability Locus $\gamma=1$

Stable slug, spherical particles $|u|=0.14$

Ellipticity

Slip velocity $|u|$
### Table 1:

<table>
<thead>
<tr>
<th>Shape</th>
<th>( N )</th>
<th>Ellipticity</th>
<th>( q )</th>
<th>( a_x )</th>
<th>( a_y )</th>
<th>( a_z )</th>
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<td>1.0</td>
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<td>0.979</td>
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<tr>
<td>Prolate Ellipsoid</td>
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<td>0.998</td>
<td>2.0</td>
<td>1.0</td>
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<td>Sphere</td>
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<td>1.0</td>
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<td>Cuboid</td>
<td>29874</td>
<td>-</td>
<td>0.956</td>
<td>4.0</td>
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### Table 2:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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<tr>
<td>Gas density, ( \rho_g )</td>
<td>1.2kg/m³</td>
</tr>
<tr>
<td>Gas viscosity, ( \eta )</td>
<td>1.8 × 10⁻⁵ Pas</td>
</tr>
<tr>
<td>Particle density, ( \rho_p )</td>
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<tr>
<td>Normal spring stiffness, ( k_n )</td>
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<tr>
<td>Tangential spring stiffness, ( k_t )</td>
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<tr>
<td>Coefficient of restitution</td>
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</tr>
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</table>

### Table 3:

- **A** Suspended grain flow over a stationary bed of particles
- **B** Suspended grain flow over a shearing bed of particles
- **C** Fully suspended dilute grain flow
- **D** Slug flow

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<table>
<thead>
<tr>
<th>Ellipticity</th>
<th>Voidage Fraction</th>
<th>Bulk Density</th>
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</thead>
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<td>0.5</td>
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<td>636.8</td>
</tr>
<tr>
<td>0.7</td>
<td>0.363</td>
<td>636.8</td>
</tr>
<tr>
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<td>0.400</td>
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<td>1.0</td>
<td>0.418</td>
<td>582.3</td>
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<tr>
<td>1.1</td>
<td>0.399</td>
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<tr>
<td>1.3</td>
<td>0.370</td>
<td>629.6</td>
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<td>1.5</td>
<td>0.368</td>
<td>631.5</td>
</tr>
<tr>
<td>Figure 1</td>
<td>Various types of super-quadric shapes.</td>
<td></td>
</tr>
<tr>
<td>Figure 2</td>
<td>Simulation set-up for pneumatic conveying in a rectangular duct. The bed is half filled and a pressure gradient is imposed on the gas to drive the particles.</td>
<td></td>
</tr>
<tr>
<td>Figure 3</td>
<td>Cross section of the particle bed at $t = 0.5s$ for spherical particles. Pressure gradients from $0.6kPa/m$ to $6.0kPa/m$ are shown. The particles are shaded by relative velocity magnitude.</td>
<td></td>
</tr>
<tr>
<td>Figure 4</td>
<td>Cross section of the particle bed at $t = 0.5s$ for prolate ellipsoidal particles with ellipticity 0.5. Pressure gradients from $0.6kPa/m$ to $6.0kPa/m$ are shown. The particles are shaded by relative velocity magnitude.</td>
<td></td>
</tr>
<tr>
<td>Figure 5</td>
<td>Bed cross section for shaped particles at a pressure gradient of $3.0kPa/m$ at steady state, shaded by relative velocity magnitude.</td>
<td></td>
</tr>
<tr>
<td>Figure 6</td>
<td>Phase diagram showing flow modes at varying particle ellipticity against pressure gradient. The applied pressure gradient is not a linear scale. Flow modes are given in table 3.</td>
<td></td>
</tr>
</tbody>
</table>
Figure 7  Average pressure and pressure gradient along particle bed, 0.6s after start up for spherical particles with pressure gradient of 2.4kPa/m. The region occupied by the slug is between the dashed vertical lines.

Figure 8  Stability diagram showing locus of stability parameter $\gamma = 1$ for slip velocity $|u_r|$ and ellipticity. Shaded region shows $\gamma > 1$, so systems with parameters within this region will form stable slugs.

Table 1  Properties of super-quadric shapes investigated.
Table 2  Computational parameters used in simulations.
Table 3  Observed pneumatic conveying flow modes.
Table 4  Voidage fraction and bulk density measured from simulations.