A conceptual framework for incorporating surface-groundwater interactions into a river operation-planning model

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Abstract

Groundwater discharge constitutes a significant proportion of the total flow volume in most rivers. The exchange flux between surface and groundwater greatly impacts the surface as well as the groundwater balance with serious implications on ecosystem health especially during low flow conditions. There is a move toward conjunctive river-aquifer management with the integration of surface-groundwater exchange fluxes into surface and groundwater models to manage water as a single resource. Groundwater-Surface water (GW-SW) exchange fluxes are seldom integrated into river operation and planning models. The time lags associated with the impacts of groundwater processes on nearby rivers can greatly compromise the forecasting capacity of river models especially during low flow conditions.

This paper presents a conceptual framework for integrating GW-SW exchange fluxes into the new generation river operation-planning model ‘Source Rivers’. The proposed GW-SW Link Module adopts a simple pragmatic approach for estimating the exchange fluxes between a river reach and the underlying aquifer using explicit analytical solutions. This flux becomes an inflow/outflow to that river reach and forms part of the routing and calibration processes. The exchange flux comprises four components: (1) natural exchange flux resulting from river stage fluctuations during low flow conditions, within bank and over bank fluctuations; (2) flux due to groundwater extraction; (3) flux due to changes in aquifer recharge; and (4) flux due to changes in evapotranspiration. The sum of those components during every time step dictates whether the river loses water to or gains water from the aquifer.

The proposed analytical solutions were found to provide flux predictions that agree favourably with those derived from a numerical groundwater model. Recognising that the simplifying assumptions that underpin the explicit analytical solution are likely to be violated in the natural world, a suite of criteria was recommended for their use under many violating conditions related to boundary conditions, head gradients, and aquifer heterogeneity. Low flow indices were adopted to demonstrate the critical role of GW-SW exchange flux when predicting river low flows. Explicit accounting of GW-SW interactions into river operation and planning models greatly enhances their forecasting capacity especially during low flow conditions.
Introduction

Significance of surface-groundwater interactions

Groundwater discharge from shallow aquifers into catchment surface waters represents the major part of the total flow volume in most rivers (Wittenberg, 2003). The magnitude and direction of the exchange flux between surface and groundwater is mainly determined by the hydraulic gradient between a river and the underlying aquifer. It can greatly impact the surface water and groundwater balance with serious implications on ecosystem health especially during low flow conditions. Krause et al. (2007) reported that although groundwater contributions from a river stretch in the northeast German lowlands represent only 1% of the annual total discharge within the river, its impact is much higher during low flow conditions in summer where 30% of the river runoff which is generated in the catchment is originated by groundwater discharge from the riparian zone along this river. During extreme low flow conditions, the groundwater-surface water (GW-SW) exchange fluxes are crucial in determining the hydro-chemical conditions and resulting ecological stress during a time which may coincide with the main vegetation growth period (Krause et al., 2007).

The critical issues of water resource availability and ecological sustainability have highlighted the need to integrate surface-groundwater interactions in both groundwater and surface water models thus leading to a conjunctive approach that manages water as a single resource. Recent initiatives by the Australian government such as the Murray-Darling Basin Sustainable Yields Project (see http://www.csiro.au/partnerships/MDBSY.html) have explicitly required the incorporation of groundwater fluxes when estimating surface water resources in the basin. In Kansas in the USA, aquifer management regulations now include baseflow when evaluating a groundwater permit application (Sophocleous, 2000). It is now recognised that in order to maintain healthy rivers and wetlands, only a small fraction of aquifer recharge can be exploited. As such, there is a move toward conjunctive river-aquifer management by amending safe yield regulation to include baseflow, which is represented as a groundwater withdrawal that has already been appropriated (Sophocleous, 2010).

Processes contributing to the groundwater-surface water exchange flux

A number of processes contribute to the exchange flux between surface and groundwater; most importantly they include: groundwater extraction, aquifer recharge (including diffuse recharge, recharge from irrigation return, and recharge from overbank flow), bank storage, and evapotranspiration. Aquifer recharge represents a gain to the GW system (which eventually enhances discharge to the river), while groundwater extraction and evapotranspiration represent a loss to the GW system (which eventually depletes the river). Bank storage is a dynamic phenomenon whereby a river recharges the aquifer during a flood event and then water discharges back to the river after the flood wave recedes. The net result of those processes at any point is space and time can either lead to a gaining or a losing river. Some or all of those processes might contribute to the exchange flux with the extent of the contribution varying significantly in space and time depending on the hydrogeological configuration as well as human and-or environmental drivers.
Drought conditions that result in limiting a surface water resource can place enormous pressure on a groundwater resource via increased groundwater extraction. Groundwater extraction, which initially depletes the aquifer, eventually depletes nearby rivers by either reducing aquifer discharge to rivers or by inducing river recharge to aquifers. Long-term sustained extraction can lead to significant reductions in river flow; Mair and Fares (2010) investigated river flow in a small Hawaiian island and reported reductions in river flow of up to 36% since 1971 as a direct result of groundwater extraction. River depletion also leads to intermittency of river flow, which has adverse ecological impacts. Stream fed aquifer recharge may be a naturally occurring phenomenon but it is enhanced by the increased downward gradients that are developed due to extensive groundwater abstraction. Andersen and Acworth (2009) analysed the annual flow difference between two gauging stations on the Namoi River in eastern Australia, which indicated that losses from the Namoi River is significantly larger than the combined surface water diversion and groundwater abstraction. Large overbank events, although not very frequent, they can lead to significant aquifer recharge. Evapotranspiration is a significant discharge mechanism for groundwater in shallow aquifers (Cook and Rassam, 2002), in closed hydrologic basins (Abdalla, 2008), and along riparian buffers. Bank storage can significantly reduce storm-inflow peaks and contributes partially to baseflow, the natural groundwater discharge to a river (Hantush et al., 2002). Exchange fluxes during bank storage can significantly affect water and nitrogen budgets in perennial, as well ephemeral streams with perched water tables (Rassam et al., 2008a).

The temporal and spatial scales at which these processes contribute to the exchange flux vary significantly. For example, river depletion due to groundwater extraction is associated with time lags that range from days to hundreds of years; the extent of the extraction activity may vary along a river reach thus leading to gaining and losing sub-reaches. Because of the intensive spatial and temporal variability there is a need for dynamic modelling of their impacts on river flow.

**Rationale for current work**

Near-river aquifer systems are complex due to the difficulties in estimating flows into and out of the aquifer, the complicated nature of the GW-SW interaction processes, and the uncertainty of aquifer properties (Sophocleous, 2010). Because of this complexity, computer models are used to model groundwater systems and estimate the exchange flux between surface water and groundwater. These fluxes are seldom integrated with river operation-management models (Valerio et al., 2010). Traditionally, the interaction between surface and groundwater is implicitly accounted for during the routing calibration of river management models. The slow time-variant nature of the groundwater processes leads to unrealised impacts that are outside the calibration period of the river model, which compromises the forecasting capacity when used outside its calibration period. Fully coupled models such as MODHSM (Hydrogeologic Inc., 1996) and GSFLOW (Markstrom et al., 2008) have the capacity to simultaneously simulate the flow of surface water, groundwater, and their interaction. However, they do not take into account the complex operational aspects of river management.

The most commonly used approach to account for GW-SW exchange in river operation and planning models is linking them to groundwater models such as MODFLOW (McDonald and Harbaugh, 1988). This can be achieved either via a
dynamic link where the models are run simultaneously (Valerio et al., 2010), or via an external link whereby fluxes estimated by the groundwater model are imported as known inputs into the river model. The latter approach has been adopted by the Murray Darling Basin Sustainable Yields Project in Australia. Due to the very strict time constraints that prevented dynamic coupling of the surface and groundwater models, the GW-SW interactions were evaluated from groundwater models using a new ‘dynamic equilibrium’ approach. A number of shortcomings were identified in this approach (Rassam et al., 2008b). Lessons learnt from the Murray-Darling Basin Sustainable Yields Project have emphasized the need and demonstrated the lack of tools that can pragmatically model the GW-SW interactions on a large scale. This paper presents the GW-SW Link Module, which accounts for the exchange fluxes between surface and groundwater in the new-generation river operation-planning model ‘Source Rivers’ being developed by the eWater Cooperative Research Centre (www.ewater.com.au/uploads/files/FL110218_SourceRivers_web.pdf).

Objectives of current work

In this paper, a conceptual framework for incorporation the exchange fluxes between surface and groundwater into a river operation-planning model is presented. The proposed GW-SW Link Module, which has been integrated into the river operation-planning model ‘Source Rivers’, explicitly accounts for the interaction between surface and groundwater. The GW-SW Link Module is an intermediate-complexity model that adopts analytical solutions to estimate the exchange fluxes between surface and groundwater with minimal data requirements. It incorporates the effects of lag times due to individual stresses then assumes linearity to estimate the cumulative impact on river flow, which is mostly significant during low flow conditions. This concept has been implemented successfully in the analytical elements modelling thus providing the correct trade-off between model complexity and the correct representation of the groundwater system (e.g., Bakker and Strack, 2003).

This paper presents the conceptualisation of the proposed model, describes the basic processes that contribute to the exchange fluxes between surface and groundwater, presents a suite of analytical solutions for evaluating the flux for various river-aquifer configurations, tests the capacity of the model to evaluate cumulative impacts by comparing its predictions to numerical simulations, then explores the applicability limits of the analytical solutions and discusses calibration issues. This study is strictly a stand-alone application of the GW-SW Link Module that involves a proof of concept. A subsequent paper will present an implementation of the GW-SW Link Module within ‘Source Rivers’ in an actual river reach and will demonstrate how the exchange fluxes estimated by the GW-SW Link Module affect the calibration and predictions of the river model.

Conceptualisation of GW-SW Link Module

Source Rivers is designed to manage water resources across rural and urban catchments, for both human and environmental uses of water, taking into account an unlimited range of future water, land use and climate scenarios. Source Rivers discretises the river system into a number of 1-dimensional links, which is usually in the order of tens of kilometres, with upstream and downstream nodes (see Figure 1A).
Along any river system, there are inflows (such as rainfall-generated inputs or irrigation returns) and losses (such as seepage under a dam or evapotranspiration). Knowing the inflow at a particular upstream gauge, the river model takes into consideration the inflows and losses along a particular reach, routes the flow with the aim of predicting outflow at the downstream node. The GW-SW Link Module adds an explicit daily term representing GW-SW exchange fluxes, which is a time-variant gain/loss that is accounted for during calibration of the river model.

The GW-SW Link Module conceptualises a groundwater aquifer that underlies the river, with either a saturated or an unsaturated connection (see cross-sectional view of the link, Figures 1B and 1C). The connection type greatly influences the active processes that contribute to the exchange fluxes between surface and groundwater; this aspect will be discussed in detail in the following sub-section. A range of river-aquifer configurations are available in the GW-SW Link Module. They are strictly aligned with those for which explicit analytical solutions for flux are available in the peer reviewed literature. Each configuration is associated with a set of different boundary conditions, aquifer layering, and river connection with the aquifer.

The exchange flux comprises the following components: (1) low-flow flux at a predevelopment condition; (2) flux due to river bank fluctuations (within and overbank); (3) flux due to changed aquifer recharge; (4) flux due to groundwater extraction; and (5) flux due to changed evapotranspiration. The GW-SW exchange flux is tracked historically and projected in a time-varying manner into the future thus accounting for the significant time lags associated with the delayed impacts of

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**Figure 1:** schematic showing; (A) 1-dimensional link model, (B) processes for unsaturated connection, (C) processes for saturated connection, and (D) conceptual cross-sectional schematic for analytical solutions
groundwater processes. This enhances the calibration of the river model where the unrealised impacts of existing stresses and any future groundwater developments are taken into consideration when running the river model in a forecasting mode.

**Underpinning assumptions**

- The river, which is represented with a 1-dimensional link, always interacts with the underlying aquifer. This interaction results in an exchange flux, which is estimated (by default) on a daily time basis.

- The interaction between the river and the underlying aquifer comprises lateral exchange that is orthogonal to the river for a saturated connection, and downward flow (river recharge) for an unsaturated connection. That is, there is no aquifer flow component parallel to the river, which implicitly means that adjacent river links do not interact with each other.

- The type of connection between a river and the underlying aquifer is assessed *a priori* for each link based on the GW-SW connectivity mapping of the region; it remains constant during the simulation period.

- If the river-aquifer connectivity varies spatially within a long river reach, then it should be divided into links with a uniform connection type. Similarly, if the user believes that the connection type is likely to change during a simulation (e.g. due to a large flood event or long-term wet/dry conditions), then the simulation should be terminated at the time at which the connection changes, then the model is re-run with the new connection type.

- A linear system is assumed, which means that a head change resulting from applying a stress to the aquifer is not large enough to alter its transmissivity.

- The assumption of linearity means that the overall GW-SW exchange flux along each link at any time step can be estimated by summing the individual fluxes arising from the active GW-SW processes within that link.

- River-aquifer configuration is constant along a link but can vary along different links. A suite of river-aquifer configurations are available.

- The hydraulic parameters of the aquifer and the river-aquifer interconnection are constant along a link (homogeneity is assumed). Heterogeneity along a long river reach can be modelled by dividing it into sub-links with different configurations and/or hydraulic parameters.

- The analytical solutions for discharge response assume that the river-aquifer system is initially at equilibrium, i.e., water level in river and nearby aquifer is represented by a straight line.

- The exchange flux resulting from within-bank river stage fluctuations are assumed to be constant along the entire link.

- The interaction between evapotranspiration and groundwater extraction is neglected.
Hydraulic connection between river and aquifer

The interaction between surface and groundwater water is largely impacted by the type of connection, which dictates the direction and magnitude of the exchange flux between the two systems. A fully saturated hydraulic connection between a river and the nearby aquifer occurs when the watertable intersects the river (Figure 1C). Under such conditions, the flux is a linear function of the head gradient between the river and the aquifer. When the groundwater table in a losing stream system drops slightly below the streambed (Figure 1B), seepage flux continues to increase linearly with small declines in the groundwater level but further declines eventually leads to the formation of an unsaturated zone below the river bed where the flux-head relation becomes non-linear. The relationship between seepage flux and depth to water table ultimately becomes asymptotic and approaches the maximum flux condition when the groundwater table becomes deep. Here, a linear relationship between flux and head gradient is assumed up to some critical depth (specified by the user) beyond which the flux become independent of depth to groundwater table. This approach is similar to that adopted in MODFLOW where the river bottom variable ‘RBOT’ represents the critical depth at which flux behaviour changes (McDonald and Harbaugh, 1988).

One still needs to differentiate between full hydraulic connection and disconnection where the latter condition prevents return flows from the aquifer (Vázquez-Suñé et al., 2007). Under such conditions aquifer stresses have minimal impacts on river flow, for example, groundwater evapotranspiration becomes irrelevant and stream depletion becomes less significant as the depth to groundwater table approaches the critical depth. The criteria proposed by Brunner et al. (2009) can be used to identify the formation of unsaturated conditions. This criteria needs to be established by the user and the decision on the type of connection be made accordingly.

For an unsaturated connection, the exchange fluxes between surface and groundwater is merely the river recharge as influenced by changes in stage height, as the depth to the groundwater table is considered to remain at the maximum critical depth. For a saturated connection, the exchange fluxes between surface and groundwater will include the fluxes originating from pre-development conditions in addition to those arising from changes in aquifer recharge, groundwater extraction, within and overbank river fluctuations, and evapotranspiration. Those processes will be discussed in detail in the following section.

Processes contributing to the GW-SW exchange flux

Natural river-aquifer interaction driven by river stage fluctuation

A river is continuously interacting with the underlying aquifer. This interaction can be discretised into three components depending on the state of flow in the river: (1) interaction during base-flow (non-event, low-flow) conditions; (2) interaction during within-bank flood events; and (3) interaction during overbank flood events. This is mathematically represented as follows:

\[ Q_n(t) = \pm Q_l(t) \pm Q_b(t) + Q_o(t) \]  

where \( Q_n \) is the natural GW-SW interaction flux (\( L^3/T \), where \( L \) represents length units and \( T \) represents time units) at any time step resulting from river-stage
fluctuation, $Q_l$ is the interaction during low-flow conditions ($L^3/T$), $Q_b$ is the interaction during within-bank fluctuations ($L^3/T$), and $Q_o$ is the interaction during over-bank fluctuations ($L^3/T$). Those three components are described in more detail herein.

1. Interaction during low (base) flow conditions

A river may continuously lose water to, or gain water from the underlying aquifer (as shown in Figure 1C); neutral cases are also possible with no head gradient and zero exchange. Considering a regional river-aquifer system that is at equilibrium (i.e. recharge into the aquifer is equal to discharge to the river system), this exchange flux would remain constant with time. It is given by:

$$Q_l = (h - h_{wt}) \times \left( \frac{K \times Y \times W}{M} \right) = \Delta h \times C$$

(2)

where $Q_l$ is the GW-SW exchange flux at low flow conditions ($L^3/T$), $h_{wt}$ is the groundwater table level (L), $h$ is the river stage level (L), $\Delta h$ is the head difference between the river stage and the groundwater table (L), $K$ is the hydraulic conductivity of the riverbed sediments (L/T), $Y$ is the length of the river link (L), $W$ is the width of the river link (L), $M$ is the thickness of the riverbed sediments (L), and $C$ is the hydraulic conductance of the river-aquifer interconnection ($L^2/T$). At steady state conditions with no climate change, no land use change, and no groundwater extraction, the long-term average $\Delta h$ should be at a state of dynamic equilibrium thus resulting in a constant average baseflow. Ideally, $\Delta h$ can be obtained from analysing long term river flow and groundwater records at pre-development conditions. It is important to note that adding this term to the flux equation precludes the need to model recharge and ET at predevelopment conditions, one only needs to model the change that has occurs past that period. Realistically, this term is implicitly accounted for during calibration of the river model (since it is constant and very difficult to evaluate).

2. Interaction during within-bank flow events (bank storage):

River stage rises as the river flow increases following rainfall events. This triggers a change or reversal in head gradient between the river and the aquifer thus resulting in water infiltrating into the aquifer. When the flood wave recedes, this water subsequently returns to the river. The significance of bank storage varies with the size and of the river floodplain and its hydraulic properties (Knight and Rassam, 2007).

The bank storage flux formulation, derived by Moench et al. (1974) and Hall and Moench (1972) as implemented by Birkhead and James (2002), is used:

$$Q_b(t) = \frac{2TY}{b} \sum_{i=2}^{\infty} \left( h_i - h_{i-1} \right) \times \sum_{n=1}^{\infty} \exp \left\{ -\left( \frac{(2n-1)\pi}{2b} \right)^2 \frac{T\Delta t}{S} \right\} \left( t - i + 0.5 \right)$$

(3)
where $Q_b$ is the bank storage flow rate ($L^3/T$) where discharge from the bank to the river is positive (consistent with the sign convention for a gaining river), $T$ is the aquifer transmissivity ($L^2/T$; $T=K h^*$, where $K$ is aquifer hydraulic conductivity and $h^*$ is average saturated aquifer thickness), $S$ is the aquifer specific yield, $Y$ is the length of the link ($L$), $h$ is the river stage height ($L$) at any step $i$, $\Delta t$ is the time step ($T$), $t$ is the time level, $n$ is the number of times the exponential term needs to be summed (pilot runs have indicated diminishing benefits in accuracy for $n>25$), and $b$ is the lateral extent of the floodplain ($L$). Note that this solution is for a finite boundary so even if one uses a semi-infinite solution for the recharge/extraction effects, the effective width within which bank storage effects are active need to be specified. As bank storage is effective on both sides of the river bank, Equation 3 needs to be implemented independently for each side of the river (with a possibility for each side having a different width). Ideally, the simulation should start after a dry period where return flow from previous bank storage processes have ceased meaning that the river-aquifer is at the low-flow equilibrium condition.

3. Interaction during over-bank flow events:

Overbank flow during major flood events results in large quantities of water that travels as sheet flow across the floodplain. After the flood-wave subsides, the overbank water returns back to the river minus the portion that had evaporated and infiltrated through the floodplain surface. Some of the overbank water may be trapped in depressions thus leading to the formation of floodplain wetlands; some of this water will evaporate and the remainder will continue to infiltrate long after the flood-wave subsides. All the infiltrated water travels through an unsaturated zone and eventually recharges the aquifer in a time variant manner. This recharge eventually discharges to the river after a time lag that depends on the orthogonal distance to the river and aquifer diffusivity. It is worthwhile noting that very large flood events may significantly raise the water table with the potential to change the connection type between the river and the aquifer. This condition should be independently assessed and in the event of it happening, then a new simulation that reflects the new connection type should be conducted.

An algorithm is proposed to calculate the evaporative flux from a floodplain wetland in addition to the infiltrative flux that passes through its bed while maintaining water mass balance. The evaporative flow rate is given by:

$$f_{ev} = A_w \times PE\quad(4)$$

where $f_{ev}$ is the evaporative flux ($L^3/T$), $A_w$ is the inundated floodplain area ($L^2$), and $PE$ is the open water surface evaporation rate ($L/T$).

The infiltrative flow rate is given by:

$$f_{inf}(t) = A_w \times K_s \times \Delta h_w(t-1)\quad(5)$$

where $f_{inf}(t)$ is the infiltrative flux ($L^3/T$), $K_s$ is the hydraulic conductivity of the wetland bed ($L/T$), and $\Delta h_w(t-1)$ is the head gradient in the wetland at the previous time step.

The wetland volume ($V_w$) and water level is calculated at the end of every time step as follows:
\[ V_w(t) = V_w(t-1) - dt(f_{ev} + f_{inf}) \]  
\[ h_w(t) = \frac{V_w(t)}{A_w} \]

where \( V_w(t) \) is the wetland volume (\( L^3 \)), \( V_w(t-1) \) is the wetland volume at the previous time step (\( L^3 \)), \( h_w(t) \) is the water head in the wetland (\( L \)), and \( dt \) is the time step (\( T \)).

The time series for \( f_{inf}(t) \) becomes a recharge time series (the time lag in the unsaturated zone is neglected in this formulation). The length of the recharge time series depends on the initial wetland volume and the rates of evaporation and infiltration, that is, the time series ends when \( V_w \) becomes zero (the wetland empties). Note that even when the wetland empties, the discharge impact continues to be estimated along the entire simulation (to account for the delayed discharge response). The discharge resulting from an instantaneous recharge source to a semi-finite aquifer \( f_{inf}(t) \) is given by (Knight et al., 2005):

\[ Q_o(t) = (f_{inf} \times dt) \frac{a}{2t\sqrt{\piDt}} \exp\left(\frac{-a^2}{4Dt}\right) \]

where \( Q_o(t) \) is the instantaneous river discharge resulting from aquifer recharge sourced from overbank flow (\( L^3/T \)), \( f_{inf}(t) \) is the instantaneous recharge sourced from overbank flow during one time step (\( L^3/T \)), \( D \) is aquifer diffusivity which is equal to \( T/S \) (\( L^2/T \)), \( a \) is the orthogonal distance from the centre of the floodplain wetland to the river (\( L \)), and \( t \) is time (\( T \)). The overall discharge is estimated by summing up the individual discharge response for every recharge value pulse using a convolution approach similar to that used in Equation 3. It is worthwhile noting that integrating Equation 8 yields the response for a step change as given by the Glover and Balmer (1954) solution for stream depletion, which will be discussed in the next section as given by Equation 9.

**River depletion due to groundwater extraction**

Groundwater extraction is one of the most important processes that impact the exchange flux between surface and groundwater water. Extraction-induced river depletion is defined as the reduction of river flow due to induced infiltration of stream water into the aquifer or the capture of aquifer discharge to the river. After a long period of uniform extraction, the cone of depression takes its final shape at steady-state. The time required to reach steady state varies linearly with aquifer diffusivity and non-linearly with the square of the orthogonal distance between the extraction-well and the river. Other important factors that may significantly affect river depletion include riverbed clogging, degree of stream partial penetration, and aquifer heterogeneity.

There are numerous analytical solutions for river depletion derived for a variety of river-aquifer configurations. Those solutions integrate the discharge response along an infinite river reach; the spatial distribution of fluxes along a river reach will be discussed in the next section. Note that the extraction rates are linear multipliers in the analytical solutions; hence, they do not affect the time scales during which the impacts of extraction occur. Modelling a variable extraction rate including decommissioning of a extraction source is simply modelled by applying a
complementary (new) source with the adequate magnitude and sign notation required for representing the new extraction rate. For example, if the extraction rate is to be increased from 500 m$^3$/day to 750 m$^3$/day then one needs to add a new source with a extraction rate of 250 m$^3$/day; similarly, if this source is being decommissioned, one needs to add a new source with a rate of -500 m$^3$/day.

The following analytical solutions representing a range of river-aquifer configurations are available for implementation within the GW-SW Link Module.

1. The Glover and Balmer (1954) solution estimates cumulative discharge response resulting from a step change in extraction rate (Figure 1D; extraction source at distance=a) in an unconfined, single-layer homogenous aquifer (Figure 1D; $\phi=0$; $H_1=H_2$), with a fully penetrating river, an infinite boundary opposite to the river (Figure 1D; $c=\infty$), where the river/aquifer systems are at initial equilibrium condition (Figure 1D; $\theta=0$).

$$Q_p(t) = P \times \text{Res}(t) = P \times \text{erfc} \left( \frac{a}{2\sqrt{Dt}} \right)$$

(9)

where $Q_p$ is the cumulative depletion flux resulting from groundwater extraction (the fraction of pumped water being sourced from the river (L$^3$/T), $P$ is the extraction rate (L$^3$/T), $\text{Res}(t)$ is the non-dimensional discharge response as a function of dimensional time, $D$ is aquifer diffusivity (L$^2$/T), $a$ is the orthogonal distance between the river and the pump (L), and $t$ is time (T).

2. The Hall and Moench (1972) solution estimates cumulative discharge response $Q_p$ in an unconfined, single-layer homogenous aquifer, with semi-infinite boundary, and a river that fully penetrates the aquifer and has a semi-pervious bed.

$$Q_p(t) = \text{erfc} \left( \frac{a}{2\sqrt{Dt}} \right) - \exp \left( \frac{a}{\alpha} + \frac{Dt}{\alpha^2} \right) \text{erfc} \left( \frac{a}{2\sqrt{Dt}} + \frac{\sqrt{Dt}}{\alpha} \right)$$

(10)

where $\alpha$ is by a retardation factor that represents the effect of a low-conductivity barrier, which was defined by Hantush (1965) as the effective thickness of aquifer required to cause the same head loss as the semi-pervious barrier; $\alpha = m K/K^*$ where $K$ is the hydraulic conductivity of the aquifer, $m$ is the width of the semi-pervious barrier, and $K^*$ is its hydraulic conductivity.

3. The Knight et al. (2005) solutions estimate cumulative discharge response $Q_p$ in an unconfined, single-layer homogenous aquifer, with a river that fully penetrates the aquifer. Two of the solutions proposed by Knight et al. (2005) are implemented in the GW-SW Link module, the first accounts for the presence of a no-flow boundary located at a distance ‘c’ opposite to the river (Figure 1D):

$$Q_p(t) = \frac{a}{2\sqrt{\pi DT}} \exp \left( \frac{-a^2}{4DT} \right) + \sum_{n=1}^{\infty} \left[ \frac{2n+1}{2\sqrt{\pi DT}} \exp \left( \frac{-2(n+1)^2}{4DT} \right) \right]$$

(11)

The second solution accounts for the presence of a head gradient in the aquifer (Figure 1D; $\theta\neq0$; $H_1=H_2$):

$$Q_p(t) = \frac{1}{2} \text{erfc} \left( \frac{a - \kappa t}{2\sqrt{D}} \right) - \frac{1}{2} \exp \left( \frac{aK}{D} \right) \text{erfc} \left( \frac{a + \kappa t}{2\sqrt{D}} \right)$$

(12)
where \( \kappa = K \tan(\theta)/S \), which is the physical velocity of a water particle down the slope, and \( \theta \) is the angle of the water table (Figure 1D).

4. The Hunt (2003) solution estimates cumulative discharge response \( Q_p \) in an semi-confined, homogenous aquifer, with semi-infinite boundary and a river that partially penetrates the aquifer. The formulation requires knowledge of the aquitard parameters including its thickness beneath the river, conductivity, porosity, and specific storage, in addition to the semi-confined aquifer transmissivity and storativity. Those parameters collapse into three non-dimensional parameters that are actually required for the solution.

\[
Q_p(t) = \text{erfc}\left(\frac{1}{2\sqrt{t}}\right) - \exp\left(\frac{\lambda}{2} + \frac{t\lambda^2}{4}\right)\text{erfc}\left(\frac{1}{2\sqrt{t}} + \frac{\lambda\sqrt{t}}{2}\right) - \lambda\int_0^t F(\alpha, t)G(\alpha, t)\,d\alpha \quad (13)
\]

Where \( \alpha \) is a variable related to aquitard thickness, permeability, and porosity, and \( \lambda \) is a variable related to aquitard permeability, its thickness below the river, and river width. The definitions of the functions \( F \) and \( G \) are lengthy and can be found in Hunt (2003).

**Change in recharge**

Various land use practices such as irrigation and farm dams lead to changed drainage below the root zone. This alters the recharge to the underlying aquifer thus leading to changing discharge to the nearby river. Recharge and groundwater extraction are two opposite processes, the former leading to increased discharge to the river and latter reducing it. However, the mathematical response function describing them are identical, the only difference is the recharge/extraction rate having opposite signs.

The recharge can either be as a step change or variable in time. For the former case, the algorithms for estimating river depletion (Glover and Balmer, 1954; Hall and Moench, 1972; and Knight et. al, 2005) may be used to estimate the discharge response resulting from a change in recharge in an unconfined aquifer. The extraction rate \( P \) is replaced by the magnitude of change in recharge rate \( \Delta R \) with an opposite sign hence resulting in positive discharge flux to the river rather than a negative depletion flux. For the latter case where recharge is highly time-variant, the algorithm used for overbank return is used with \( f_{\text{inf}}(t) \) of Equation 8 replaced by \( \Delta R(t) \); the convolution approach demonstrated in Equation (3) is then used to estimate the total flux resulting from any number of recharge pulses.

**Change in evapotranspiration**

Evapotranspiration (ET) is a major component of the water budget in vegetated areas that have relatively shallow water tables. In such areas, transpiration directly from groundwater by near-shore vegetation can intercept base flow that would otherwise discharge to a stream. Depending on the positioning of the root zone with respect to the water table, the plant can extract water directly from groundwater, from the unsaturated zone, or from both. Actual evapotranspiration remains equal to the potential (PET) value as dictated by the climate down to some depth called the ‘transition depth’ where ET starts to shift from atmospheric-control to soil-moisture control (Rassam and Williams, 1999). This implies that below this depth, soil properties and vegetation type would dictate the magnitude of actual ET. At some
depth called the ‘extinction depth’, evapotranspiration becomes very low as the vegetation can no longer extract any groundwater. The ET-decline function proposed by Shah et al. (2007) is adopted in this work and is given by:

$$\text{ET}_a = \frac{\text{ET}}{\text{PET}} = 1 \quad \text{for} \quad d \leq d'$$  \hspace{1cm} (14a)

$$\text{ET}_a = \frac{\text{ET}}{\text{PET}} = e^{-g(d-d')} \quad \text{for} \quad d > d'$$  \hspace{1cm} (14b)

where $\text{ET}_a$ is the actual evapotranspiration rate (L/T), PET is the potential evapotranspiration rate (L/T), $d$ is the depth to groundwater table (L), $g$ is a decay coefficient (T$^{-1}$), and $d'$ is the transition depth (L) where ET shifts from atmospheric control to soil-moisture control. Shah et al. (2007) has recommended estimates of the fitting parameters for various land uses and soil types which have been incorporated in the GW-SW Link Module. The total flow rate due to ET is given by:

$$Q_{ET} = Y \times b \times \text{ET}_a$$  \hspace{1cm} (15)

where $Q_{ET}$ is the ET flux (L$^3$/T), $Y$ is the length of the link (L), $b$ is the lateral extent of the floodplain (L), and $\text{ET}_a$ is actual evapotranspiration rate defined by Equation 14. Extreme care should be taken to ensure that there is no double accounting for the impacts of any process. It is emphasized that the pre-development flux base flow implicitly accounts for pre-development ET fluxes. Therefore, one only needs to account for changes in actual ET that have occurred since that time due to land clearing or re-vegetation during the post-development stage.

**Spatial and temporal aspects of GW-SW interactions**

**Spatial and temporal distribution of fluxes along river reach**

The spatial distribution of GW-SW exchange fluxes vary with every process. GW-SW exchange driven by within-bank changes of river stage is assumed to occur instantaneously along an entire link. However, all other contributors to the flux (recharge, extraction, and ET) are point-source aquifer stresses that are spatially defined by their x-y coordinates. The x-coordinate represents the distance ‘a’, which appears in all the analytical solutions (e.g., Equations 8 and 9). The analytical solutions for discharge response integrate fluxes along an infinite river reach. Since the link used in this model has a finite length, one needs to apportion fluxes along this finite length. This issue becomes important for aquifer stresses that are close to a node separating two adjacent links whereby a stress located within one link may contribute to the flux in the adjacent link. The flux can be apportioned as follows (Rassam et al., 2004):

$$f(y,t) = \frac{a}{\pi(y^2 + a^2)} \exp\left(-\frac{y^2 + a^2}{4Dt}\right)$$  \hspace{1cm} (16)

where $f$ is the instantaneous flux response at any lateral distance from a node per unit length of the river (L$^3$/T) at time (t), $a$ and $y$ are the x-and y-coordinates of the recharge/extraction source relative to the river link (L) as shown in Figure 1A, D is...
aquifer diffusivity \((L^2/T)\), and \(t\) is time \((T)\). To obtain the total flux, Equation 16 needs to be integrated along the entire length of the link.

**Lumping of similar aquifer stresses**

The individual impact of each aquifer stress (recharge/extraction) is added to yield the cumulative impact of a number of stresses. Hence, model run-time becomes a function of the number of active stresses in the aquifer, which can be in the order of hundreds for long river reaches. The discharge response for recharge/extraction is a function of the hydrological response time \(a^2/D\) (Knight et al., 2005). One can effectively lump the impacts of ‘hydrologically similar’ aquifer stresses thus reducing the run time. Any number of aquifer stresses located anywhere within a river reach but having identical schedules and similar hydrologic response times can be lumped into one entity having a single response function. In this case, the recharge or extraction rates for all sources are added then multiplied by the response function to yield the overall contribution to the flux at each time step.

**Smart time stepping**

Source Rivers by default operates on a daily time step. The GW-SW Link Module also estimates the exchange fluxes between a river and the nearby aquifer on a daily basis. Aquifer stresses (extraction/recharge) having a very slow discharge response (due to low \(D\) or large \(a\)) may lead to very marginal variations in the daily fluxes. For such slow-response processes, the user has the option to extend the time step of the GW-SW Link Module into weekly or monthly to optimise model run times by reducing the number of mathematical operations. Non-dimensional analysis can be adopted to smartly arrive at an optimum time step that results in a minimal acceptable error. When the dimensional time scale is normalised with respect to the hydrological response time, one obtains a dimensionless time scale \(\tau = t/(a^2/D)\). When the response \(\text{Res}(t)\) given by Equation (9) is plotted versus dimensionless time \(\tau\), a unique characteristic response function is obtained. Since this response is a function of aquifer diffusivity and the distance between the stress location and the river (‘\(a\’\) and ‘\(D\’\) both known quantities), one can predict \textit{a priori} the change in discharge response during a groundwater time step and hence recommend a longer time step during which a noticeable response occurs (e.g., 1%). The slope of the steepest section of the non-dimensional discharge response curve is evaluated then \(\Delta \tau\) corresponding to one percentile change in response is identified. Substituting the known values of ‘\(a\’\) and ‘\(D\’\), the actual time (time step, \(\Delta t\)) required to arrive at the 1% change in discharge response is determined. One can reduce the response during a time step to increase accuracy, for example halve the response to 0.5%, since the relationship is linear \(\Delta t\) will be halved too. The model then uses linear interpolation between successive groundwater time steps to calculate the daily response. Note that this is a pre-processing procedure where the model only recommends a groundwater time step (based on a 1% response change) and the user has the option to implement it or adhere to the daily time step.
Calculating total exchange fluxes

Unsaturated connection

For an unsaturated river-aquifer connection, the exchange flux for groundwater depths equal to or deeper than the critical depth is given by:

\[ Q_{un}(t) = \left[ h(t) - h_{cr} \right] \times C \tag{17} \]

where \( Q_{un} \) is the GW-SW exchange flux for an unsaturated connection (L\(^3\)/T), \( h_{cr} \) is the critical groundwater level below which the flux-head relationship becomes independent of depth to groundwater table (L), \( h \) is the river stage level (L), and \( C \) is the hydraulic conductance of the river-aquifer interconnection (L\(^2\)/T) given by Equation (2).

Saturated connection

The linearity of the governing equations that underpins the analytical solutions means that the principle of superposition is applicable, which allows the summation of individual impacts to obtain their overall accumulation. For example, the total stream depletion flux resulting from a number of extraction points across the aquifer is estimated as follows:

\[ Q_p(t)_{total} = \sum_{i=1}^{n} Q_p(a, t) \tag{18} \]

where \( Q_p(t)_{total} \) is the total depletion flux for a number of extraction sources (L\(^3\)/T), \( Q_p(t) \) is the cumulative depletion flux (L\(^3\)/T) up to time \( t \) for an individual extraction source resulting from a step increase in extraction rate (e.g., given by Equation 9), and \( n \) is the number of active sources along a link. A formulation similar to Equation 16 is implemented for multiple recharge sources to yield their total impacts \( Q_R(t)_{total} \).

The overall river-aquifer exchange flux at any time is conceptualised as the summation of the fluxes resulting from all the active GW-SW interaction processes at that time, which is given by:

\[ Q_{sw-gw}(t) = \pm Q_n(t) - Q_{ET}(t) - Q_p(t)_{total} + Q_R(t)_{total} \tag{19} \]

where \( Q_{sw-gw} \) is the overall river-aquifer exchange flux (L\(^3\)/T) at time \( t \), \( Q_n(t) \) is the natural interaction flux (L\(^3\)/T) due to river stage fluctuations given by Equation (1), \( Q_p(t)_{total} \) is the total interaction flux (L\(^3\)/T) due to groundwater extraction, \( Q_R(t)_{total} \) is the total interaction flux (L\(^3\)/T) due to changes in recharge, and \( Q_{ET}(t) \) is the interaction flux (L\(^3\)/T) due to changes in evapotranspiration.

Modelling experiment

A comprehensive modelling experiment was carried out to test the validity of the proposed conceptualisation to model GW-SW interactions within the river operat-
planning model ‘Source Rivers’. As mentioned earlier, this experiment is a stand-alone application of the GW-SW Link Module, which involves testing the proposed analytical approach and comparing its results to numerical predictions derived from MODFLOW. The modelling experiment covers four aspects: (1) suitability of the analytical solutions to model multiple stresses and processes, (2) spatial and temporal distribution of fluxes along the river, and (3) significance of GW-SW interactions during low flow conditions, and (4) applicability of the analytical solutions. Details of the modelling experiment are listed in Table 1 with reference to the conceptual models shown in Figures 1 and 2.

The suitability of the analytical solutions to model multiple stresses and processes was tested by applying three extraction sources and a time-variable constant head boundary condition representing river stage fluctuations using stage heights derived from the flow time series for the Namoi River shown in Figure 3. The flow domain contains two hydraulic conductivity zones, which serve two purposes: firstly, to represent heterogeneity along the river, and secondly, to demonstrate the concept of lumping sources that have similar hydrological response time. Referring to Table 1 (Simulation 1), the hydrological response time ($a^2/D$) for both $S_1$ and $S_2$ is equal $(15.625)$. Hence, those two sources that have extraction rates of 500 and 300 m$^3$/day, respectively, are lumped into one source with an extraction rate of 800 m$^3$/day. A variable extraction rate was used for $S_3$ to demonstrate the concept of implementing a negative source to model a reduction in extraction rate. The analytical results are then compared to results from the MODFLOW simulations. This part of the modelling experiment also aims at validating the concept of linearity and superposition.

![Figure 2: Conceptual model for MODFLOW simulations](image)
Table 1: Details of modelling experiment (refer to Figures 1 and 2 for conceptual models and parameter symbols)

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Type</th>
<th>Details</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Flux response due to multiple sources and processes</td>
<td>Analytical</td>
<td>Knight et al. (2005) for finite domain Equation 3 for bank storage</td>
<td>$D_1 = 7840 \text{m}^2/\text{day}; D_2 = D_3 = 4000 \text{m}^2/\text{day}; a_1 = 350 \text{m}; a_2 = 250 \text{m}; a_3 = 1000 \text{m}; c = 3000 \text{m}$; extraction rates shown below</td>
</tr>
<tr>
<td></td>
<td>Numerical</td>
<td>$X = 3 \text{km}; Y = 10 \text{km}; Y_1 = 5 \text{km}; X_{s1} = 350 \text{m}; Y_{s1} = 2500 \text{m}; X_{s2} = 250 \text{m}; Y_{s2} = 7500 \text{m}; X_{s3} = 1000 \text{m}; Y_{s3} = 8000 \text{m}$</td>
<td>$K_1 = 10 \text{m/day}; K_2 = 19.6 \text{m/day}; S = 0.05; h = 20 \text{m}$; extraction rates $S_1 = 500 \text{m}^3/\text{day}; S_2 = 300 \text{m}^3/\text{day}; S_3 = 500 \text{m}^3/\text{day}$ then reduced to $200 \text{m}^3/\text{day after 200 days}$</td>
</tr>
<tr>
<td>2. Spatial and temporal distribution of fluxes along the river</td>
<td>Analytical</td>
<td>Equation 16</td>
<td>$D = 1,000 \text{m}^2/\text{day}; a = 1000 \text{m}$</td>
</tr>
<tr>
<td></td>
<td>Numerical</td>
<td>$X = 3 \text{km}; Y = 10 \text{km}; Y_2 = 0$</td>
<td>$K_1 = 5 \text{m/day}; h = 10 \text{m}; S = 0.05; a = 1000 \text{m}$; Zone Budgets at reach widths $a_2, 4a_2, 8a_2, 12a_2, 16a_2, and 20a_2$</td>
</tr>
<tr>
<td>3. GW-SW interactions during low flows</td>
<td>Analytical</td>
<td>Knight et al. (2005) for finite domain</td>
<td>$D = 6000 \text{m}^2/\text{day}; a_1, 2, 4 \text{ km}$; Daily flux response deducted from river flows (stage heights derived from flows (Figure 3))</td>
</tr>
<tr>
<td>4. Effect of no-flow boundary</td>
<td>Analytical</td>
<td>Compare Glover and Balmer (1954) to Knight et al. (2005) for finite domain</td>
<td>non-dimensional analysis with $c = 2a, 4a, 6a,$ and $9a$; upper and lower bound envelops with $0.5D$ and $2D$</td>
</tr>
<tr>
<td>5. Effect of head gradient</td>
<td>Analytical</td>
<td>Compare Glover and Balmer (1954) to Knight et al. (2005) with head gradient</td>
<td>$a = 100 \text{m}, S = 0.05$; variable $K$ and $h$ to arrive at $D = 9,000$ and $18,000 \text{m}^2/\text{day}$; head gradient $\tan \theta$ and $h$ varied to arrive at $h/\tan \theta = 100; 250; 600; 3,000$; and $10,000$.</td>
</tr>
</tbody>
</table>
| 6. Effect of aquifer heterogeneity | Analytical | Cases 1-3: use Knight et al. (2005) with an effective aquifer thickness $h^*$ based on knowledge of prevailing heads  
Case 4: Hall and Moench, 1972 | Case 1: $K = 2 \text{ m/day}; h^* = 20 \text{m & 28m}; S = 0.1; a = 300 \text{m}$  
Case 2: $K = 2 \text{ m/day}; h^* = 20 \text{m & 24m}; S = 0.1; a = 3,500 \text{m}$  
Case 3: $H_1 = 38 \text{m}; H_2 = 90 \text{m}; h^* = 51.5 \text{m}; a = 500 \text{m}$  
$K = 0.5 \text{m/day}; S = 0.05; \phi = -3^\circ$  
Case 4: $D = 2,000 \text{m}^2/\text{day}; a = 2,000; \alpha = 100; 1000; 5,000; 10,000$ |
| 6.a Cases 1-2: variable saturated aquifer thickness due to a groundwater mound | Numerical | Cases 1-2: $X = 25 \text{km}; Y = 0$  
Case 3: $X = 1,000 \text{m}; Y = 50 \text{m}; h = 0$  
Case 4: $X = 25 \text{km}; Y = 50 \text{m}; h = 0$ | Case 1: $K_1 = 2 \text{ m/day}; h_1 = \text{variable}; S = 0.1; a = 300 \text{m}$  
Case 2: $K_1 = 2 \text{ m/day}; h_1 = \text{variable}; S = 0.1; a = 3,500 \text{m}$  
Case 3: $K_1 = 0.5 \text{ m/day}; h_1 = 90 \text{m}; S = 0.05; a = 500 \text{m}$  
Case 4: $K_1 = 10 \text{ m/day}; h_1 = 10 \text{m}; S = 0.05; a = 2,000 \text{m}; \alpha = 100; 1000; 5,000; 10,000$ |
| 6.b Case 3: variable saturated aquifer thickness due to sloping base aquifer | Analytical | $H_1 = 38 \text{m}; H_2 = 90 \text{m}; h^* = 51.5 \text{m}; a = 500 \text{m}$ | $K = 0.5 \text{m/day}; S = 0.05; \phi = -3^\circ$ |
| 6.c Case 4: presence of semi-pervious riverbank | Analytical | Case 4: Hall and Moench, 1972 | $K = 0.5 \text{m/day}; S = 0.05; \phi = -3^\circ$ |

Unless otherwise indicated, $H_1 = H_2; \theta = \phi = 0$; Equilibrium initial conditions exist between river and aquifer; $h_i$ is initial head in numerical model.
The spatial and temporal distribution of fluxes along the river was analytically modelled using Equation 16. The flux was apportioned along the river reach at steady state and then the results were compared to those obtained from the Zone Budget package of MODFLOW (Table 1, Simulation 2).

The significance of GW-SW interactions during low flow conditions is highlighted by using flow data from the Namoi catchment in eastern Australia. The Namoi River catchment is a tributary of the Murray-Darling River Basin (see Figure 4), which has experienced a consistent increase in the level of groundwater extraction in recent times. The Namoi River is hydraulically connected to the shallow aquifer system with two general zones of interaction. Upstream of Boggabri the Namoi River tends to lose water to the aquifer whereas downstream of Boggabri it tends to gain water from the aquifer. The exchange fluxes along the reach from Keepit Dam to Narrabri are in the order of ±(10-15) ML/day where ± refers to gains and losses to and from the river (McNeilage, 2006). River flows from the Boggabri gauge on the Namoi River for the period April/1992 to December/1994 were used (Figure 3). It includes a drought period during which groundwater extraction significantly increased by about 10,000 ML/annum for the management zone upstream of Boggabri (McNeilage, 2006). A hypothetical groundwater extraction zone is activated from April/1992 until July/1994 (just prior to the drought period). An extraction rate of 27.4 ML/day was implemented, which is equivalent to the 10,000 ML/annum rise in groundwater extraction during the drought period. Three scenarios were investigated where the distance between the hypothetical source and the river was 1 km, 2 km, and 4 km (Table 1, Simulation 3). The flux response for each case was deducted from the daily flows, then two low flow indices were derived, namely, the 90th percentile flow ($Q_{90}$) and the number of zero-flow days.

Figure 3: Flow time series and flow duration curve for Namoi River at Boggabri
A series of modelling simulations were carried out to test the applicability of the various analytical solutions implemented in the GW-SW Link Module. The experiment aims at assessing the accuracy of the analytical solutions as their underpinning assumptions may be violated in the natural world. Those violations may arise from the following effects: (a) presence of a no-flow boundary; (b) presence of head gradients; and (c) aquifer heterogeneity that may arise from a number of factors such as a variable saturated aquifer thickness (due to the formation of groundwater mound or due to a sloping base aquifer), the presence of a semi-pervious river bank, or a heterogeneous river reach. Details of this part of the modelling experiment are covered in Simulations 4-6, Table 1. The groundwater mound (numerical simulation 6a, Table 1) is simulated in MODFLOW with an aquifer having a uniform initial saturated aquifer thickness of $h_i=20\text{m}$. Subsequently, an irrigation development is applied 500m away from the river covering an area $300\times300\text{m}$ with a recharge rate of $120\text{mm/yr}$. The output of a 33-year simulation is then used as an initial condition for two new simulations with two recharge sources applied individually at the mound peak and behind it, respectively. The semi-pervious river bank was modelled by including a vertical layer having a lower hydraulic conductivity next to the river (see Figure 1D; Simulation 6c, Table 1).

Results

Flux response from multiple sources and processes

Figure 5 shows that summing the individual impact of each source yields a cumulative impact on the river, which is in excellent agreement with estimates obtained from the numerical MODFLOW model. In this simulation, the concept of lumping multiple sources having similar hydrological responses has also been demonstrated (where $P_1$ and $P_2$ were lumped into one source with a combined extraction rate of $800\text{m}^3/\text{day}$). It has been shown that the reduction in extraction rate can be modelled by adding a new
source with a negative rate (P4 with -300m$^3$/day was added to P3 to yield a net rate of 200m$^3$/day).

Figure 5: Exchange flux due to one process; several extraction sources

Figure 6 shows the GW-SW exchange fluxes due to the combined impacts of groundwater extraction and bank storage. Note that the extraction layout for this example is similar to the previous one with the exception of using a homogeneous aquifer having a hydraulic conductivity of 10 m/day. The simulation results confirms that the analytical solution provide adequate estimates for the flux with an error margin of up to 4%, which tends to stabilise after 800 days. This error margin is consistent with the findings of Birkhead and James (2002) who reported that Equation 3 may underestimate the bank storage flux by up to 5%.

Figure 6: Exchange flux due to two processes; extraction and bank storage
Spatial and temporal distribution of fluxes along the river

Figure 7 shows that at early times, most of the flux is sourced from a narrow band of the river reach opposite to the source. The influence length increases with time and is function of aquifer diffusivity; it asymptotically approaches infinity at steady state. The total GW-SW exchange flux along a finite length along the river (e.g., link length) is estimated by integrating the area under the curve. At steady state, the shape of the curve becomes independent of aquifer diffusivity as the exponential term of Equation 16 equates to unity at infinite time. Plotting the product of instantaneous response and distance ‘a’ versus the normalised distance ‘y/a’ results in a unique relationship that describes the flux distribution along the river at infinite time (Figure 8). For example, integrating the flux over a distance y=2a (1a on either side of the source) shows that exactly 50% of the extraction/recharge is sourced from a narrow 2a-wide stretch of the river reach (see insert Figure 8). Performing several integrations at various distances shows how the flux response asymptotically reaches unity with excellent agreement to numerical simulations using MODFLOW and the Zone Budget package (see insert Figure 8).

Figure 7: Flux distribution along a river at various times

Figure 8: Non-dimensional characteristic flux distribution along a river at steady state
An example application of the concept of apportioning flux is demonstrated here at a steady condition with reference to Figure 8. Assume two equal river reaches [a] and [b] of length L=35 km that are defined by Nodes 1, 2, and 3 where Node 3 is the downstream node. A recharge source has coordinates a=5 km and y₁=25 km (following the convention used in Figure 1; note that y₂=L- y₁). By definition, the source is located at the origin y/a=0. In non-dimensional terms (y/a), Nodes 3, 2 and 1 are located relative to the source at y₁=5, y₂=-2, and y₃=-9, respectively. The flux that is apportioned to Reach [1] is represented by Area 1 whereas the flux apportioned to Reach [2] is represented by the sum of Areas 2 and 3. The areas can be derived from insert Figure 8, which provides integrals for the area on both sides of the origin (e.g., y/a=2 includes an area that starts from y/a=–1 and ends at y/a=+1 and accounts for 0.5 of the flux; from symmetry, the area on either side of the origin is half the total area).

Therefore, Area 3=0.88/2=0.44 (where 0.88 is derived from insert Figure 8 for y/a=10; as area extends from origin to y₁=5, it is halved to yield the area on one side of the source). Area 2=0.72/2=0.36 (where 0.72 is derived from insert Figure 8 for y/a=4 as y₂=2). The flux ratio apportioned to Reach [2] is equal to (Area 2+Area 3) = 0.44+0.36=0.80. The flux apportioned to Reach 1 stretches from y/a=-9 to -2; it can be estimated by finding the total area from y/a=0 to y/a=-9 then subtracting from it Area 2. Therefore, area from y/a=0 to -9 = 0.92/2=0.46 (where 0.92 is derived from insert Figure 8 for y/a=18 as y₃=9). The flux ratio apportioned to Reach [1] is equal to 0.46-0.36=0.10. Note that the total flux ratio for the two reaches adds up to 0.90, which leaves 0.10 most of which belongs to the reach downstream of Node 3. The flux ratio apportioned during every time step of the model run is estimated by implementing Equation 16 and performing a numerical integration to estimate the area under the curve, which increases with time (as shown in Figure 7).

Significance of GW-SW interactions during low flow conditions

It is very important to emphasize that the impacts of the interactions between surface and groundwater become paramount during low flow conditions. The flow time series for the Namoi River at Boggabri (see Figure 4) indicates that under normal climate conditions (period prior to 30/6/1994) the 50th and 90th percentile flow is 265 ML/day and 35.5 ML/day, respectively (see insert Figure 4). The GW-SW exchange flux constitutes about 3.8% of the median flow, which is an insignificant amount that could well be within the uncertainty limits of the gauge data. However, the GW-SW exchange flux constitutes about 30% of flow during low flow conditions. This contribution would be much more significant under drought conditions (see Figure 4 encircled data 1995); this data shows a significant drop of more than two orders of magnitude in low flows, which is a direct result of increased groundwater extraction. The groundwater extraction during 1995 was double the average rates during the period between 1987 and 1993 (McNeilage, 2006). The GW-SW Link module is used to conduct a modelling exercise to highlight this phenomenon using the Namoi river flow data. Note that the intention of this hypothetical exercise is not to replicate observed flows but merely to highlight the impact of groundwater extractions on river low flows.
Results in Figure 9 show a significant decline in $Q_{90}$ and an exponential rise in the number of zero flow days due to increased groundwater extraction. The results highlight the critical role of the distance between the extraction source and the river. The flux response to groundwater extraction varies with the square of the distance between the extraction point and the river. During the 825-day simulation, the impacts of the extraction seem to diminish when the extraction source is 4 km from the river. Hence, knowing the time scales within which the impacts of groundwater extractions occur, one can allow additional extractions that are distant enough to cause minimal impact on the river during a specific planning period.

**Applicability limits for analytical solutions**

A set of assumptions are usually required to idealize (simplify) the flow problem in order to arrive at an explicit analytical solution for every river-aquifer configuration. In order to obtain accurate results from the analytical solutions, their underpinning assumptions should not be violated. Since the most basic solution for estimating discharge response was introduced by Glover and Balmer in 1954, a number of advancements have been made to incrementally add versatility (and hence complexity) to the solution. More complexity leads to more parameters and higher uncertainty, which eventually defies the purpose of having a simple model. Therefore, one always endeavours to use the simplest model that can produce acceptable results knowing that the simplifying assumptions may be violated to some extent under natural flow conditions thus resulting in some errors.

It is reasonable to postulate that the magnitude of the error is directly related to the extent to which the assumptions have been violated. Here, the applicability limits of various analytical solutions used in the GW-SW Link Module are individually tested and compared to the basic Glover and Balmer (1954) solution as well as results obtained from MODFLOW numerical models. Subsequently, criteria for the application of the solutions are identified for three classes of violations relating to: (1) boundary conditions, (2) head gradients, and (3) aquifer heterogeneity.
Effect of no flow boundary

Most analytical solutions for discharge response assume a semi-infinite flow domain meaning that the flow boundary opposite to the river is infinite. One would expect that if the distance ‘a’ from the stress source to the river is much smaller than the distance to the no-flow boundary ‘c’, the infinite boundary solution should produce adequate results. On the other hand, if the two distances are comparable there would be a significant discrepancy. Knight et al. (2005) proposed a solution for a finite flow domain, which accounts for the effect of a no-flow boundary. However, the other solutions (e.g., Hall and Moench (1972) that accounts for a low-conductivity layer) still assume an infinite boundary opposite to the river. Hence, the effect of a no-flow boundary needs to be investigated.

Figure 10 compares results from the Glover and Balmer (1954) with those from Knight et al. (2005). The y-axis in Figure 10 represents the flux response (Res(t) in Equation 9) whereas the x-axis represents non-dimensional time (time normalised by $a^2/D$). It shows that for $c>9a$, the two solutions produce identical results within the flux response up to 90%. On the other hand, the effect of the no-flow boundary becomes pronounced when $c=2a$. To put matters into perspective, the magnitude of the error resulting from violating the boundary condition is compared to that resulting from a $\pm 100\%$ uncertainty in knowledge of aquifer diffusivity, this is a conservative uncertainty envelope as this parameter is usually varied by an order of magnitude (Marsily et al., 2005). Figure 11 shows that when $c>6a$, the effects of the no-flow boundary can be neglected.

Effect of head gradients

The analytical solution for discharge response assumes initial hydrologic equilibrium between the river and the aquifer whereby the aquifer water table is horizontally aligned with the river water level. Under natural conditions, most rivers are either gaining or losing and hence have head gradients to or away from the river, respectively. A downward (positive) head gradient towards the river speeds the discharge response whereas a reverse gradient slows it down. The mathematical formulation of Knight et al. (2005) for sloping base aquifers with a head gradient...
applies to both gaining and losing rivers where only the sign notation of the angle needs to be reversed. Here, the sensitivity of the discharge response with respect to a positive head gradient is tested by comparing results from Glover and Balmer (1954) to Knight et al. (2005). In this modelling experiment, the aquifer thickness and head gradient are varied but the diffusivity kept constant. Another trial was carried out with half the diffusivity to test the model sensitivity to it.

The sensitivity analysis has shown that head gradient (represented by the slope of the water table \(\tan \theta\)) and aquifer thickness (h) are the two critical parameters contributing to the discrepancy between predictions obtained from the two solutions with the ratio \(h/\tan \theta\) as an indicator. Figure 11 shows that the discrepancy varies with time and reaches a maximum at a response of about 70%. The Glover and Balmer (1954) solution exhibits a maximum error of about 10% when \(h/\tan \theta \approx 600\). It produces excellent results when this ratio approaches 3000, which is equivalent to a very commonly occurring combination of a head gradient of 1% and an aquifer thickness of 30m. Results have demonstrated that the magnitude of the diffusivity per se neither affects the errors pattern nor its magnitude (Figure 11, see equal-length dotted arrows at 60\% response for D=9000 and 18000 m\(^2\)/year). However, Figure 11 also shows that the Glover and Balmer (1954) solution grossly underestimates discharge response for shallow aquifers with steep head gradients (e.g., \(h/\tan \theta = 100\)).

**Effect of aquifer heterogeneity**

Variable saturated aquifer thickness: the saturated thickness of an aquifer may vary spatially either due to a sloping base or due to the formation of a groundwater mound or a depression cone resulting from a recharge or an extraction source, respectively.

Firstly, the effect of a groundwater mound is investigated. The new irrigation development (Simulation 3a, Table 1) has resulted in a groundwater mound with a peak of 30m, which is 50\% higher than the initial aquifer thickness of 20m thus...
significantly adding to aquifer transmissivity. Figure 12 shows that using the initial saturated thickness of 20m in the analytical solution (at both locations in front and behind the groundwater mound, Case-1 and Case-2, respectively), results in underestimating the response by about 5%. If one can independently estimate the head distribution after the build-up of the mound, which can be done analytically, a modified saturated aquifer thickness can be derived to improve the predictions of the analytical solution. Figure 12 shows that using such an effective thickness that accounts for increased transmissivity resulting from the presence of the mound results in excellent agreement with MODFLOW predictions (h=28m and 24m for Case-1 and Case-2, respectively).

Secondly, the effect of a sloping base aquifer is investigated. Figure 12 (Case-3) shows that using the same approach that accounts for a non-uniform aquifer thickness improves the performance of the analytical solution and yields excellent agreement with the numerical results.

Semi-pervious river bank: Figure 13 shows that when the retardation factor $\alpha$ (which accounts for the presence of the semi-pervious layer) is as low as 100, the semi-pervious layer has virtually no effect and the response is identical to that provided by the basic Glover and Balmer (1954) formulation (marked no retardation in Figure 13). Note that $\alpha=100$ means a 1-m semi-pervious layer having a hydraulic conductivity that is two orders of magnitude lower than that of the aquifer. For higher retardation factors, the response starts to be significantly affected by the semi-pervious layer where the Hall and Moench (1972) formulation continues to provide adequate predictions for $\alpha$-values of up to 1,000. However, as $\alpha$ approaches 5,000, the Hall and Moench (1972) solution underestimates the response by a maximum of 10% at early times but this discrepancy diminishes at large times (compared to MODFLOW predictions).
Aquifer heterogeneity along river reach: As outlined in the underpinning assumptions, a river link is assumed to be homogeneous with constant hydraulic parameters. Hantush et al. (2002) indicated that heterogeneity of a river-aquifer system can be accounted for efficiently by dividing the river reach into homogenous segments (sub-reaches), the outflow from one segment becomes the inflow to the next segment. This approach is adopted here to represent heterogeneity. The validity of this approach will be demonstrated in the next section.

Model calibration

Models achieve their best predictive capacity via a calibration process whereby their parameters are optimised to provide predictions that in best agreement with field observations. Groundwater models are usually calibrated against observed pressure head observations and the calibrated model is then used to provide predictions for GW-SW exchange fluxes (e.g., Zone Budget at the river boundary for a MODFLOW model). Assuming sound aquifer conceptualisation and good knowledge of various stresses to the system, aquifer diffusivity (D) and riverbed conductance (C) are the main parameters that are optimised during the calibration of a groundwater model. A river model on the other hand, is calibrated against observed flow data at a downstream gauge. The traditional calibration of a river model such as IQQM (Simons et al., 1996) includes: (1) flow routing parameters (varies with routing scheme); (2) a loss function (losses as a function of river flow that includes overbank losses and river recharge to the underlying aquifer for losing river systems); and (3) a residual flow time series (inflows including ungauged inputs and gains from the groundwater for gaining river systems).

As the new generation ‘Source Rivers’ model includes both surface and groundwater processes, its calibration would entail concepts from both the surface water and groundwater systems. The calibration would still be against observed flow data at a downstream gauge with two additional groundwater parameters, namely, aquifer diffusivity and riverbed conductance. However, one can independently calibrate the groundwater parameters and keep them constant (or implement bounds to their
variability) during calibration of the river model to control the non-uniqueness problem. This can be done outside of ‘Source Rivers’ using analytical solutions for the application of a flood wave response to estimate aquifer diffusivity and river conductance using the groundwater levels of a floodplain observation well (Ha et al., 2007).

An important issue that needs to be highlighted here is the timing of the delayed impacts of the groundwater processes and how they relate to the calibration period of the river system model. Referring to Figure 14, one can assume a pre-development era when the coupled surface-groundwater system was at equilibrium thus resulting in a steady-state exchange flux, $Q_{pd}$. Groundwater extractions would upset this state of equilibrium resulting in a new exchange flux that is time-variant. With a high level of groundwater development, one would expect this to be the status quo almost everywhere. Therefore, the calibration period of a river model would most likely coincide with a groundwater system that is at a transient state where the flux, during part or the entire calibration period, is time-variant. The traditional calibration of river models such as IQQM implicitly accounts for this time-variant GW-SW exchange flux. However, when the model is operated in a forecasting mode, it would disregard the effect of the unrealised impacts of existing developments as well as the impacts of future developments.

The new generation ‘Source Rivers’ model overcomes this problem as it explicitly accounts for the interaction between surface and groundwater via the GW-SW Link Module. To achieve a realistic calibration that results in a model with a strong forecasting capability, one must historically track all the changes in aquifer stresses and model their impacts prior to, and during the calibration period of the river model ($t_0$-$t_1$-$t_2$ in Figure 14) and continue to account for their unrealised impacts during a subsequent forecasting simulation ($t_2$ to $t_3$ in Figure 14). Note that the pre-development flux will most likely be an unknown quantity that is implicitly accounted for in the calibration of the river model (which maintains mass balance).

![Figure 14: Schematic showing relation of GW-SW exchange fluxes to calibration and forecasting periods for river model](image-url)
Conclusions

The exchange flux between surface and groundwater can greatly impact the surface water and groundwater balance with serious implications on ecosystem health especially during low flow conditions. River models implicitly account for the interaction between surface and groundwater where the exchange fluxes become part of unaccounted gains/losses during a black-box calibration process. Explicit accounting for the interaction between surface and groundwater greatly enhances the forecasting capacity of river models especially during low flow conditions. In this paper, a conceptual framework for incorporating surface-groundwater interactions into the river operation-planning model ‘Source Rivers’ was presented. The Groundwater-surface water (GW-SW) Link Module explicitly accounts for the interaction between surface and groundwater. It adopts explicit analytical solutions to evaluate the exchange flux due to individual processes, then, uses the concept of linearity and superposition to estimate the overall exchange flux between the surface and groundwater systems. The following may be concluded:

- The total exchange flux between a river and the underlying aquifer comprises the following components: (1) natural exchange flux due to river stage fluctuations, which is the sum of three components, exchange during baseflow conditions, exchange during within-bank fluctuations, and exchange during overbank fluctuations; this component assumes a river-aquifer system at dynamic equilibrium, with all stresses present accounted for in the next three components; (2) exchange flux due to groundwater extraction; (3) exchange flux due to change in recharge rate; and (4) exchange flux due to changes in evapotranspiration. The total exchange flux estimated by the GW-SW Link Module is incorporated into the routing scheme of the river operation-planning model ‘Source Rivers’ during every time step of a simulation.

- The applicability limits for the analytical solutions used in the GW-SW Link Module were tested as their simplifying underpinning assumptions may be violated under natural river-aquifer conditions. The following criteria for their application were recommended: (1) the effect of a no-flow boundary on the flux response becomes marginal as the distance from the river to a no-flow boundary becomes larger than six times the distance from the recharge/extraction source to the river; (2) the effect of a head gradient becomes marginal as the ratio of aquifer thickness to head gradient approaches 3,000 but it can lead to serious errors for steep gradients and shallow aquifers as the ratio becomes as low as 100; (3) the effect of aquifer heterogeneity resulting from a variable saturated aquifer thickness may be accounted for by estimating aquifer diffusivity based on an effective aquifer thickness that accounts for the non-uniform saturated aquifer thickness; (4) the effect of a semi-pervious layer can be adequately modelled using an analytical solution that implements a retardation factor ($\alpha$) with excellent results for $\alpha<1000$ and an error of about 10% as $\alpha$ approaches 5000; and (5) aquifer heterogeneity along the river can be modelled by discretising the river reach into homogenous sub-reaches.

- The concept of linearity and superposition was validated by comparing predictions of analytical solution to those obtained from MODFLOW numerical models. It was shown that the adopted analytical solutions are capable of predicting the overall
exchange flux resulting from multiple processes by adding the impacts of individual processes with errors not exceeding 4%.

- The flux distribution resulting from a recharge/extraction source located at a distance ‘a’ follows a normal distribution pattern where 50% of the flux is sourced from a reach length equal to 2a. Results of the analytical solution were found to be in very close agreement with numerical MODFLOW simulations. This functionality can be used to apportion fluxes to two adjacent links when a recharge/extraction source is located close to the node that separates them.
- It was emphasised that the interaction between surface and groundwater is paramount during low flow conditions. Explicit accounting of GW-SW interactions in river models greatly enhances their forecasting capacity during low flow conditions.
- To achieve a realistic calibration that results in a river model with a credible forecasting capability, one must historically track all the changes in aquifer stresses and model their impacts prior to, and during the calibration period. When the model is use in a forecasting mode, the remaining unrealised impacts of existing stresses, and any new stresses should be included in the future forecasting simulation.

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